An Introduction to F-Theory GUT Phenomenology

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Abstract

This review provides a pedagogical introduction to the framework of F-theory. We begin by discussing the origins of F-theory in Type IIB string theory and outline the details of elliptic fibrations as used in F-theory compactifications. We examine many aspects of F-theory GUT phenomenology in the context of a local $SU(5)$ model. Mechanisms for GUT breaking, SUSY breaking, and their implications in F-theory are discussed. Finally we present recent work on flavour physics in F-theory models.

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Chapter 1

Introduction

Ever since Maxwell demonstrated in the late 1800s that electricity and magnetism were different manifestations of the same underlying structure unification has become a prize that theoretical physics continually strives for. The unification of two previously distinct theories represents a considerable step forward in understanding and efforts will continue as long as our best theories admit a factorisation. Modern particle physics is based on the framework of quantum field theory and relies on the principle of gauge invariance. In the modern formalism electromagnetism is described by quantum electrodynamics (QED) which is a $U(1)$ gauge theory with electrons charged under the $U(1)$. QED is a remarkably successful theory but does not account for nuclear physics.

1.1 Standard Model

In order to include the weak and the strong nuclear forces the picture had to be extended and would eventually lead to the Standard Model. The weak nuclear force, responsible for β-decay and related process, was well modelled at low energies by the Fermi theory. It was known however that this could not be the final picture because it was non-renormalizable and so did not a have a consistent picture at high energies. Through the work of Weinberg, Salam, Glashow, and others in the 1970s it was discovered that if the weak force was combined with electromagnetism a renormalizable theory could be formulated. The resulting gauge theory was base on the $SU(2) \times U(1)$ gauge group. The strong interaction is also well described by quantum chromodynamics (QCD), a gauge theory based on $SU(3)$ colour symmetry with quarks transforming in the fundamental representation. So we have a gauge theory based on the gauge group

$$G_{SM} = SU(3) \times SU(2) \times U(1), \tag{1.1}$$

which today is known as the Standard Model of particle physics.

In order to make contact with the low energy physics of everyday life some of the symmetry must be broken. If it remained unbroken then there would be extra long range forces from the massless gauge bosons of the $SU(2)$ factor that were inconsistent with experiment. The framework of spontaneous symmetry breaking, already known to condensed matter physicists, was applied to break the electroweak theory to electromagnetism

$$SU(2) \times U(1) \rightarrow U(1)_{em}, \tag{1.2}$$

where $U(1)_{em}$ is the QED gauge group and comes from the diagonal component of $SU(2) \times U(1)$ and the original $U(1)$ factor is known as weak hypercharge. This symmetry breaking endows the additional gauge bosons with a mass via the Higgs mechanism. The mass lifts these W and Z bosons from low energy spectrum. The Higgs mechanism breaks the symmetry by giving a vacuum expectation value (vev) to the scalar Higgs field that becomes massive. The Higgs boson then became the most sought after prediction of spontaneous electroweak symmetry breaking and, after much effort, the international particle physics collaboration at the LHC experiment at CERN on July 4th 2012 announced the discovery of a boson consistent with the Standard Model Higgs.

This all sounds marvellous (and it is) but the job of the theoretical physicist is still not complete. There remains some problems with the Standard Model as a final theory. Most obvious is the absence of gravity. While gravity is too weak to play a significant role in particle physics experiments its absence is still unsatisfactory.

Another famous shortcoming is what is known as the hierarchy problem. In the modern approach to renormalization due to Wilson, we consider a quantum field theory as an effective theory valid only up to a certain energy scale. For the Standard Model this scale is the electroweak scale set by the Higgs mass. It is naively expected that the Higgs mass would receive large quantum corrections and push this scale up the Planck scale. The question of why there is such a large difference between the electroweak scale and the Planck scale is the hierarchy problem of particle physics.

There are other issues that are aesthetically displeasing. As we have mentioned, if a theory admits a factorization as (1.1) does, then there is always a urge to unify further into a simple group. The representations of (1.1) that chiral matter fits into are even less appealing:

$$\begin{array}{ccccccccc} (1,2)_{-\frac{1}{2}} & \oplus & (1,1)_{-1} & \oplus & (3,2)_{\frac{1}{6}} & \oplus & (\bar{3},1)_{\frac{2}{3}} & \oplus & (\bar{3},1)_{-\frac{1}{3}} \\ L = \begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix} & & e^-_R & & Q = \begin{pmatrix} u^a_L \\ d^a_L \end{pmatrix} & & u^a_R & & d^a_R \\ \text{left-handed} & & \text{right-handed} & & \text{left-handed} & & \text{right-handed} & & \text{right-handed} \\ \text{leptons} & & \text{electron} & & \text{quarks} & & \text{up quark} & & \text{down quark} \end{array}$$

With the Higgs field transforming as $(1,2)_{\frac{1}{2}}$. The notation $(R, R')_q$ denotes a SM representation with hypercharge, q, and $SU(3)$ and $SU(2)$ representations R and R' respectively. The index $a = 1, 2, 3$ are colour indices. These representations are far from elegant and they also come in three sets called generations. Much effort has been put into distilling the Standard Model into a more elegant and unified form.

1.2 MSSM and GUTs

One of the most promising solutions to the hierarchy problem is supersymmetry. Supersymmetry is a non-trivial extension of the Poincaré symmetry of spacetime that allows for fermionic directions parameterized by Grassman numbers in addition to the usual bosonic directions. This avoids the Coleman-Mandula theorem by extending the Poincaré algebra to a Lie superalgebra (a $\mathbb{Z}_2$ graded Lie algebra). The minimal $\mathcal{N} = 1$ form of this symmetry predicts that each boson (fermion) of the Standard Model will have a fermionic (bosonic) superpartner differing only through their spin quantum number. Because fermionic and bosonic loops in Feynman diagrams contribute with an opposite sign fermionic loops can cancel bosonic loops. The loop corrections to the Higgs mass are therefore suppressed, solving the hierarchy problem. The resulting theory is known as the Minimally Supersymmetric Standard Model (MSSM). It contains an $\mathcal{N} = 1$ chiral superfield for each chiral matter field of the Standard Model and two Higgs fields known as Higgs up, H_u, and Higgs down, H_d. In addition there are vector superfields for the gauge bosons. The Higgs mechanism generates masses for the chiral fermions through the Yukawa couplings in the MSSM superpotential

$$W_{MSSM} \supset \lambda^u_{ij} H_u Q^i u^j_R + \lambda^d_{ij} H_d Q^i d^j_R + \lambda^l_{ij} H_d L^i e^j_R. \tag{1.3}$$

The MSSM also hints at a more unified gauge group at higher energies. If the gauge couplings of MSSM are allowed run to higher energies with renormalization group flow they unify at $\sim 10^{16}$ GeV (Figure 1.1). This suggests that the electroweak and strong forces, which we view as distinct at energies near the electroweak scale, are actually same force at high energy/small distances. As we decrease the energy/increase the length scale this unified force fragments and appears to us as two separate phenomena.

It was known in the 1970s that the Standard Model could be embedded into a larger simple group. The unified force is then mediated by the gauge fields of this simple group. The simplest of these is based on $SU(5)$ and is known as the Georgi-Glashow model

$$SU(3) \times SU(2) \times U(1) \subset SU(5). \tag{1.4}$$

Already an attractive picture it becomes even more appealing when the chiral matter are packaged into $SU(5)$ representations. The complicated form of Standard Model representations reduces to:

$$\begin{array}{ccc} \bar{\mathbf{5}}_{\mathbf{m}} & \oplus & \mathbf{10}_{\mathbf{m}} \\ (d_R^a, L) & & (Q, u_R^a, e_R^a) \end{array}$$

with the Higgs up and Higgs down transforming in the $\mathbf{5_H} \oplus \bar{\mathbf{5}}_{\mathbf{H}}$ as $(H_u, T_u) \oplus (H_d, T_d)$. This introduces Higgs triplets, T_i, that must be remove from the low energy spectrum. The Yukawa couplings of (1.3) are then summarised in the interactions

$$\bar{\mathbf{5}}_H \times \bar{\mathbf{5}}_m \times \mathbf{10}_m \qquad \text{and} \qquad \mathbf{5}_H \times \mathbf{10}_m \times \mathbf{10}_m. \tag{1.5}$$

This elegant picture is spoiled however by the inability to match with experimental constraints for proton decay. There are potentially several operators that can mediate proton decay in these models and we will discuss them in the context of F-theory GUTs later. Experimental bounds currently set the proton lifetime at $> 2.1 \times 10^{29}$ years [1] which has historically been a challenging obstacle for four dimensional GUT model building. We must also consider the problems of GUT breaking, SUSY breaking, doublet-triplet splitting, and flavour physics

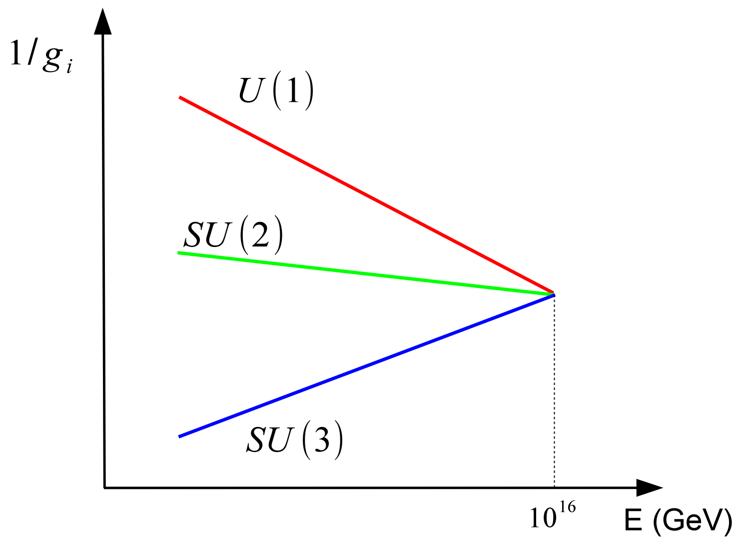


Figure 1.1: The unification of running coupling constants of MSSM at 10^{16} GeV hints at a Grand Unified Theory at this energy.

which constrain models further. We will discuss each of these in turn in the context of F-theory during this review.

There also exists non-minimal GUT models based on $SO(10)$, E_6, and other groups. In fact it is argued in [2] that physical grand unified theories should be based on exceptional groups. We can already see a pattern of embeddings from the groups we have mentioned:

$$\begin{array}{ccccccccc} E_3 \times U(1) & \subset & E_4 & \subset & E_5 & \subset & E_6 & \subset \ldots \\ SU(3) \times SU(2) \times U(1) & \subset & SU(5) & \subset & SO(10) & \subset & E_6 & \subset \ldots \end{array} \tag{1.6}$$

This sequence is illustrated by sequentially removing nodes for the E_8 Dynkin diagram (Figure 1.2).

1.3 String model building

If we want to include the effects of gravity then the current best hope is string/M-theory, the only known self-consistent theory of quantum gravity. The gravita-

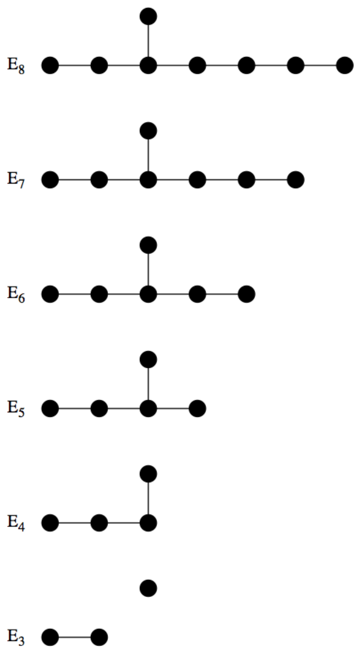


Figure 1.2: The Standard Model and GUT groups obtained as embeddings in E_8. Each diagram is obtained from that above it by removing the right-most node. Taken from [2].

tional field is then produced as a mode of a closed loop of string. There are 5 different string theories in 10 dimensions as well as M-theory in 11 dimensions. These theories are all manifestations of the same underlying theory and are interconnected via a web of dualities. Type IIA and Type IIB string theories have $\mathcal{N} = 2$ supersymmetry whereas Type I, Heterotic $SO(32)$, Heterotic $E_8 \times E_8$ strings, and M-theory have $\mathcal{N} = 1$ supersymmetry. The extra dimensions, required for Lorentz invariance of the quantum theory, must be compactified leaving the four usual spacetime dimensions in order to make phenomenologically realistic theories. There are many different possibilities for compactification which leads to hugely varying four dimensional vacua. The plethora of 4D vacua that arise from string theory has become known as the landscape. While this richness provides model builders with a lot of freedom so they can, for example, ensure proton decay is highly suppressed, it also calls into question whether the theory can be truly predictive. In order to focus in on the most promising string theory backgrounds various phenomenological constraints are imposed. Historically, the most successful approaches have been compactifications of the Heterotic $E_8 \times E_8$ string and Type IIB orientifold backgrounds with D-branes.

Type IIB orientifold model building considers a compactification of the Type IIB string theory on a Calabi-Yau threefold, X, that is quotient by an orientifold action $\Omega(-1)^{F_L}\sigma$. Here Ω is the world sheet parity operator that reverses the orientation of strings, σ is a space-time reflection of X, and the $(-1)^{F_L}$ factor is required to preserve supersymmetry. The fixed points of the orientifold action, O3/O7-planes, carry negative tension and negative RR-charge. Consistency of the quantum theory (tadpole cancellation) requires the net RR charge to be zero so positively charged D3/D7-branes must be added by hand. We will see later that adding 7-branes by hand can significantly affect the geometry but this effect is usually ignored in this setup. The degrees of freedom of strings stretched between the branes and O-planes can yield the classical groups $SU(N)$, $SO(2N)$, and $Sp(N)$ (see appendix A for a discussion of gauge theories on branes in perturbative Type II strings) that can accommodate the Standard Model. In the perturbative setup however exceptional groups are impossible so GUT model building is more challenging. A nice feature of this setup is that the gauge degrees of freedom governing particle physics are in the open string sector localised on the brane while the gravitational degrees of freedom are in closed string sector and therefore propagate in the entire compactification space. The volume of the compact manifold can therefore be tuned to alter the separation between particle physics and the Planck scale.

The Het $E_8 \times E_8$ setup is attractive because of the presence of exceptional groups. These groups appear as a result of requiring anomaly cancellation that constrains the rank of the gauge group. The physical origin of the gauge group is less clear that in the Type IIB case. The major drawback of this setup is that both the gravity and the gauge degrees of freedom lie in the open string sector so there is no natural way to separate the GUT scale from the Planck scale as in the local model picture in Type IIB orientifolds.

F-theory, a non-perturbative extension of the Type IIB picture, can accommodate both of the attractive features of these two approaches simultaneously. That is, we can have local models that separate particle physics from gravity while non-pertubative effects also allow us to have exceptional groups at our disposal. In addition to this, the F-theory framework also automatically takes care of tadpole cancellation and 7-branes are not plugged in by hand so the geometry does not get distorted. We also find that striving for a satisfactory phenomenology from F-theory compactifications can be very constraining. As a result, the F-theory corner of the landscape of vacua can become predictive.

1.4 Organization of this review

The objective of this review is to introduce F-theory as a framework for string model building and provide some applications to particular GUT models. In chapter 2 we will motivate the need for F-theory and introduce its basic building blocks. In chapter 3 we will discuss the various ingredients of an F-theory model based on an $SU(5)$ GUT. In chapters 4 we will discuss mechanisms for GUT breaking and SUSY breaking in F-theory models. Finally in chapter 5 we will discuss F-theory's approach to flavour and neutrino physics.

Chapter 2

Basics of F-Theory

In this chapter we will introduce the origins and basic ingredients of F-Theory. Through examining the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of the Type IIB supergravity and 7-brane monodromies we will motivate F-theory as a 12-dimensional non-perturbative string theory with $[p,q]$ 7-branes. We will briefly introduce F-theory as a dual description of the strongly coupled Type IIB, M-theory, and the Heterotic string. Finally, we will discuss how chiral matter and Yukawa interactions arise naturally as complex codimension 2 and 3 loci of brane intersection respectively.

2.1 Type IIB string and SL$(2,\mathbb{Z})$

The low energy limit of the closed string sector of the Type IIB superstring theory, Type IIB supergravity, has the following field content:

Bosonic:	graviton, $g_{\mu\nu}$	dilaton, ϕ	self-dual 4-form, C_4^+
	NSNS 2-form, B_2	RR 2-from, C_2	axion, C_0
Fermionic:	$2\times$ gravitini, $\psi_{\mu\dot{\alpha}}$	$2\times$ dilatini, ψ_α	

This describes a chiral theory with $\mathcal{N} = (2, 0)$ supersymmetry. The bosonic field content can be repackaged in a convenient way by defining fluxes [3]

$$\begin{aligned} &H_3 = dB_2, \quad \tau = C_0 + ie^{-\phi}, \quad G_3 = F_3 - \tau dB_2, \\ &\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge dB_2 + \frac{1}{2}B_2 \wedge F_3, \qquad F_p = dC_{p-1}. \end{aligned} \tag{2.1}$$

Here τ is known as the axio-dilaton and will be referred to throughout. The axio-dilaton encodes the IIB string coupling through $g_s = e^{-\phi}$. With these definitions we may write the bosonic action of Type IIB in the Einstein frame as

$$\begin{aligned} S_{IIB} = \frac{2\pi}{\ell_s^8}\bigg(\int d^{10}x\sqrt{-g}R - \frac{1}{2}\frac{\partial\tau\bar{\partial}\tau}{(\mathrm{Im}\tau)^2} + \frac{1}{\mathrm{Im}\tau}G_s \wedge *G_3 \\ + \frac{1}{2}\tilde{F}_5 \wedge *\tilde{F}_5 + C_4 \wedge H_3 \wedge F_3\bigg). \end{aligned} \tag{2.2}$$

In order for variation of this action to yield the correct equations of motion we require the additional restriction that $\tilde{F}_5$ be self-dual i.e. $\tilde{F}_5 = *\tilde{F}_5$. The action was written in this way in order to make manifest the $\mathrm{SL}(2, \mathbb{Z})$ symmetry where the axio-dilaton, τ, transforms in fundamental representation, the NSNS and RR 2-forms transform as a doublet, and the self-dual 4-form is invariant,

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} aC_2 & bB_2 \\ cC_2 & dB_2 \end{pmatrix}, \quad C_4 \to C_4, \quad ad - bc = 1. \tag{2.3}$$

Choosing $a = b = d = 1$ and $c = 0$ corresponds to the shift $\tau \to \tau + 1$, which is just the usual gauge invariance of the axion field. We can however also have $a = d = 0$ and $b = -c = 1$ which induces the strong/weak coupling duality $\tau \to -1/\tau$ of Type IIB. Since the NSNS and RR 2-forms transform as a doublet this transformation will exchange them which maps the F1-string to the D1-string.

The two transformations shown here are the same as the symmetries of the complex structure of a 2-torus, T^2. If we take $z \in \mathbb{C}$ and consider a parallelogram in the complex z plane with vertices $z = 0, 1, \tau, \tau{+}1$, then z is a complex coordinate of a T^2 if we identify opposite sides of the parallelogram. $\mathrm{SL}(2, \mathbb{Z})$ transformations of this parallelogram then builds up a lattice of which the parallelogram is the fundamental domain. In this way the T^2 is self-homeomorphic under $\mathrm{SL}(2,\mathbb{Z})$ transformations. In this setup τ is the complex structure modulus of the torus.

If more general $\mathrm{SL}(2,\mathbb{Z})$ transformations are considered we can form a $\binom{p}{q}$-string of which the F1 and the D1 are the special cases $\binom{1}{0}$ and $\binom{0}{1}$ respectively [4]. A $\binom{p}{q}$-string can then be thought of as a bound state of p F1 strings and q D1 strings which is BPS for p and q coprime (otherwise we have a multiple string solution). We will see later that it is the existence of these non-pertubative objects that give rise to the phenomenologically interesting exceptional gauge groups in F-theory.

2.2 7-brane backreaction and monodromies

When considering string theory backgrounds with D-branes the standard approach in the perturbative regime is to use the probe approximation. Since D-branes have non-zero tension and RR charge, they backreact on the geometry and the form fields in the ambient space. The probe approximation takes this backreaction to be negligible sufficiently far from the brane. This is common practice is physics and seems reasonable. It can be argued for by considering the Poisson equation for a source that is point-like in $n = 9 - p$ spatial dimensions i.e a p-brane,

$$\Delta\Phi(r) \sim \delta(r) \quad \Rightarrow \quad \Phi(r) \sim \frac{1}{r^{n-2}}, \qquad n > 2. \tag{2.4}$$

We therefore see that for sources of codimension $n > 2$, the backreaction asymptotes to zero as is familiar, for example, from the physics of point particles in 4 dimensions. Conversely, in the case of codimension 2 objects, like the 7-branes of our 10 dimensional string theory, this argument is not necessarily valid.

If we examine the 10 dimensional Gauss Law for the 7-brane as the electric source of the 8-form potential we find [3],

$$d * F_9 = \delta(z - z_0). \tag{2.5}$$

Here we have introduced the complex coordinate $z = x_8 + ix_9$ labelling the two real dimensions in which the brane is point-like, and we have considered a brane located at the point $z = z_0$. Integrating and using Stokes' theorem we may write

$$1 = \int_{\mathbb{C}} d * F_9 = \oint_{S^1} *F_9 = \oint_{S^1} F_1 = \oint_{S^1} dC_0, \tag{2.6}$$

where S^1 encircles the point z_0. This shows that as we encircle the 7-brane the axion field shifts $C_0 \to C_0 + 1 \Rightarrow \tau \to \tau + 1$. This is known as a monodromy. The monodromy discussed here is due to encircling a regular D7-brane. We also have to consider the monodromy of encircling $[p, q]$-branes (defined as the objects on which $\binom{p}{q}$-strings can end). These branes will introduce an $\mathrm{SL}(2, \mathbb{Z})$ monodromy that also mixes and shifts the B_2 and C_2 fields. This may be troubling because it appears our fields are multivalued. We are saved however by the $\mathrm{SL}(2, \mathbb{Z})$ invariance of the theory which we can use to undo the effect of the monodromy. In fact, every $[p, q]$-brane can locally be transformed to a D7-brane using this $\mathrm{SL}(2, \mathbb{Z})$ symmetry. This may not be done *globally* however so a generic background necessarily includes $[p, q]$-branes that cannot simultaneously be brought into D7 form.

To determine the asymptotic effect of the 7-brane on the axio-dilaton we may solve (2.6) near the 7-brane to give [5]

$$\tau(z) = \frac{1}{2\pi i} \ln\left(\frac{z - z_0}{\lambda}\right) + \ldots \tag{2.7}$$

Where the ... represents higher order terms neglected near the brane, and λ is modulus encoding the overall scale of τ. We note that at the position of the brane $\tau \to i\infty$ (this will be important later when we interpret singularities of τ as the position of a 7-brane). The presence of the logarithm indicates a severe backreaction. It is shown in [5] that this leads to an asymptotically flat space with a deficit angle. This long range effect clearly cannot be neglected in general. The same affect is seen for other codimension 2 objects known as cosmic strings.

Examining (2.7) we can identify the point $z - z_0 = \lambda$ as the source of the monodromy. At this point we have $e^{-\phi} = g_s \to \infty$. This does suggest the possibility however of entering a weak coupling limit where we choose λ to be large and focus on the region near the brane where $|z - z_0| \ll |\lambda|$. In this limit the backreaction is locally negligible. If we want to study a background with generic values of the axio-dilaton that varies over space time we must take this strong coupling point into consideration[1]. It is by accounting for the contributions of these special points that F-theory can be viewed and limit of Type IIB that is inherently strongly coupled.

2.3 F-Theory and elliptic fibrations

In this section we discuss the implication of realising the $\mathrm{SL}(2, \mathbb{Z})$ symmetry as a consequence of the geometry of spacetime. This will lead us to the framework of F-theory and elliptic fibrations. Understanding these elliptic fibrations is key in constructing phenomenological models so we will spend some time discussing them.

2.3.1 A geometric origin to the SL$(2, \mathbb{Z})$ symmetry

In section 2.1 we compared the axio-dilaton with the complex structure of a torus, and the $\mathrm{SL}(2, \mathbb{Z})$ symmetry of Type IIB supergravity with the modular invariance of this torus. It is tempting to take this correspondence seriously and look for a

[1]It is worth noting here that there does exist a construction [6] where it can be arranged that charges of the seven branes are cancelled locally by orientifold 7-planes. This setup yields a g_s that is constant over space time and can be fixed to the weak coupling regime, $g_s \ll 1$. We forgo discussion of this here in favour of more generic situations.

physical origin for the torus. This was first understood through a duality of Type IIB with M-theory in nine dimensions [4]. In this picture Type IIB is compactified on an S^1 and compared with M-theory on T^2. The $\mathrm{SL}(2,\mathbb{Z})$ is then interpreted as the symmetry of this T^2.

In [7] Vafa proposed that there should be a geometric interpretation in 10 dimensions without the need to resort to the M-theory duality in 9 dimensions. With this in mind he introduced a 12 dimensional theory with the extra 2 dimensions contained in an auxiliary torus. The 2-forms of Type IIB could then be thought of as originating from a 3-form in 12 dimensions that is reduced on the two distinct non-trivial one-cycles of the torus. The varying of the axio-dilation is then captured by the varying of the shape of these extra dimensions. We can therefore describe the spacetime as an elliptic curve (torus) fibred over the 10 dimensional spacetime of Type IIB: $T^2 \to \mathcal{M}_{10}$. The corresponding 12 dimensional supergravity has has a (10,2) signature and so is quite mysterious. The Type IIB 10 dimensional supergravity is obtained via a null reduction of the 12 dimensional theory. The absence of a fully Lorentz invariant supergravity with signature (11,1) in 12 dimensions might be an indication that the spacetime is not 12 dimensional in the usual sense. Another hint at this point is the lack of a 3-form and a 1-form in Type IIB that would arise from reduction of the 3-form on a point and on the full T^2 respectively. We will see that indeed the additional two dimensions are on a slightly different footing to the usual 10 dimensional spacetime.

2.3.2 Dualities and compactification

We have motivated the need for F-theory by considering a generically varying axio-dilaton in Type IIB. F-theory is therefore often discussed in this language and can be thought of as non-pertubative formulation of the Type IIB string. We have, however, also mentioned a connection to M-theory. To make this more precise we consider compactifying 11-dimensional supergravity (the low energy limit of

M-theory) on a $\mathcal{M}_9 \times T^2 = \mathcal{M}_9 \times S^1_A \times S^1_B$. We use the familiar S-duality relations to recover weakly coupled Type II A on $\mathcal{M}_9 \times S^1_B$ by taking the limit of vanishing radius of $S^1_A : R_A \to 0$. We then T-dualise along the remaining S^1_B to obtain Type IIB on $\mathcal{M}_9 \times \tilde{S}^1_B$. Here the radii of the circles obey $R_B \sim \frac{1}{\tilde{R}_B}$. Finally, we take the limit $R_B \to 0$ to decompactify the $\tilde{S}^1_B$ and yield Type IIB on $\mathcal{M}_{10}$. We can repeat this procedure fibre-wise to yield F-theory on $T^2 \to \mathcal{M}_{10}$. The axio-dilaton is given by the complex structure of this torus (roughly $\tau \sim i\frac{R_A}{R_B}$). We note that the Kähler modulus, A, that measures the area of the T^2 has no meaning in F-theory because we are taking the limit of $A \to 0$. This confirms our earlier suspicion that the extra two dimensions that arise in F-theory are not quite comparable to the other ten dimensions of spacetime since the volume decouples from the physics. Instead we should view them as a book keeping tool that tracks of the variation of the axio-dilation. In summary, we can think of F-theory as the theory dual to M-theory on a vanishing torus

$$F = M|_{A(T^2)\to 0}. \tag{2.8}$$

This duality can be useful to determine the degrees of freedom and effective action of F-theory. For example, the M-theory 3-form, C_3, decomposes as

$$C_3 = \tilde{C}_3 + B_2 \wedge dx + C_2 \wedge dy + B_1 \wedge dx \wedge dy, \tag{2.9}$$

where dx, and dy are representatives for the de Rahm cohomology $H^1_{dR}(T^2)$. After decompactification ($R_B \to 0$) $\tilde{C}_3$ contributes to self-dual 4-from of Type IIB, $C_4^+ = \tilde{C}_3 \wedge dy$, B_2 and C_2 become the NSNS and RR 2-forms respectively, and the components of B_1 become the off-diagonal components of the metric, g_{iy} [8].

Another duality that has been exploited for model building is that of F-theory with the heterotic string. Experience with model building using heterotic strings provides clear waypoints for the F-theory framework. Introduced in [7], the duality maps F-theory on elliptic $K3 : T^2 \to \mathbb{P}^1$ to the heterotic string on T^2. We will see later however that for some cases important for phenomenology no heterotic dual exists.

In order to get a phenomenologically viable theory it is necessary to compactify to 4 dimensions. The resulting four dimensional effective theory then depends on details of the compactification. We will assume that we have $\mathcal{N} = 1$ supersymmetry in 4 dimensions. This condition requires that the compact space be a Calabi-Yau manifold. There has been extensive study of compactifications of 10 dimensional string theories on Calabi-Yau complex threefolds, but in our case we require an elliptically fibred Calabi-Yau complex fourfold.

In most interesting setup for phenomenology we have $\mathcal{M}_{12} = \mathbb{R}^{3,1} \times Y$, and $Y : T^2 \to B_3$. Here B_3 is the three complex dimensional orientifold of the corresponding Type IIB theory. We require Y to be Calabi-Yau but not B_3. This background contains 7-branes that span all of $\mathbb{R}^{3,1}$ and wrap a four-cycle in B_3. Our four dimensional effective field theory will therefore live on the 7 brane and gauge degrees of freedom will come from the open string sector. The details of the physics is encoded in Y so in the next section we will spend some time introducing the techniques of algebraic geometry used to study elliptic fibrations.

2.3.3 Weierstrass form for elliptic fibrations

It will be instructive to first consider an elliptically fibred $K3$ so we can use the mathematical formalism developed to describe this. An elliptic curve can be described as a hypersurface in 3-complex dimensional weighted projective space $\mathbb{P}_{2,3,1}$ subject to the equivalence

$$(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z). \tag{2.10}$$

The defining equation for the elliptic curve can then always be *locally* written in the Weierstrass form

$$P_W = y^2 - x^3 - fxz^4 - gz^6 = 0. \tag{2.11}$$

As a check we can compute the number of degrees of describing the elliptic curve: 3 complex (6 real) of $\mathbb{P}^3$ - 1 complex (2 real) from (2.10) - 1 complex (2 real) from (2.11) = 1 complex (2 real) $\Rightarrow$ geometry of elliptic curve encoded in two real parameters f and g. We can relate f and g to τ through the modular invariant j-function

$$j(\tau) = \frac{4(24f)^3}{4f^3 + 27g^2} \tag{2.12}$$

where

$$j(\tau) = e^{-2\pi i\tau} + 744 + \mathcal{O}(e^{2\pi i\tau}). \tag{2.13}$$

When the shape of the torus is allowed to vary over the base space f and g become polynomials, of degree 9 and 12 respectively, of the local coordinate patch of the base space. To describe a fibration over the entire base space the patches are glued together and f and g become sections of a line bundle, $\mathcal{L}$, over the base. The non-triviality of the fibration is therefore encoded in the non-triviality of this line bundle. We note here that Calabi-Yau condition for the hypersurface is that the degree of the defining polynomial equals the sum of the weights in the equivalence relation [8]. Here we have a degree 6 polynomial and weights $2+3+1=6$, so we have a Calabi-Yau one-fold: the T^2. In order to see that the full space is Calabi-Yau we would have to take the Weierstrass form for the full fibred $K3$ rather than the reduce form we have taken in a coordinate patch (see e.g. [8]).

We saw in section 2.2 that $\tau \to i\infty$ at the postion of the seven brane. Now that we are interpreting the τ as the complex structure of the elliptic fibre we should therefore be interested in the points where the fibre degenerates. For a hypersurface defined by $P = 0$ this corresponds to the degeneration of its tangent space, $dP = 0$ [3]. To find singularities in the elliptic we apply this to the Weierstrass form (2.11). To make life easier we using (2.10) to set $z = 1$. Differentiating with respect to y and setting the result to zero we find that $y = 0$. Plugging this back into (2.11) with $z = 1$ we find

$$P_W = x^3 + fx + g = (x - a_1)(x - a_2)(x - a_3) = 0 \tag{2.14}$$

where the a_i are the roots of the cubic polynomial. Further imposing $dP_W = 0$ for this result we find

$$(x - a_1)(x - a_2) + (x - a_1)(x - a_3) + (x - a_2)(x - a_3) = 0. \tag{2.15}$$

We therefore find that the condition for the fibre to become singular reduces to satisfying $y = 0$, (2.14), and (2.15). This means that at least two of the a_i coincide. The coincidenc of roots of a polynomial results in the vanishing of the discriminant, Δ. For (2.14) we have

$$\Delta = 4f^3 + 27g^2. \tag{2.16}$$

We see then from (2.12) that

$$j(\tau) = \frac{4(24f)^3}{\Delta}. \tag{2.17}$$

Singularities of the fibre are therefore encoded in the degree 24 equation: $\Delta = 0$. We therefore have in general 24 distinct singularities in the fibre which we will label by coordinates $z_i, i = 1, ..., 24$ in the base. Near one of these zeros the j-function behaves like

$$j(\tau) \sim \frac{1}{z - z_i} \tag{2.18}$$

which, through (2.13) yields

$$\tau(z) \sim \frac{1}{2\pi i} \ln(z - z_i) \tag{2.19}$$

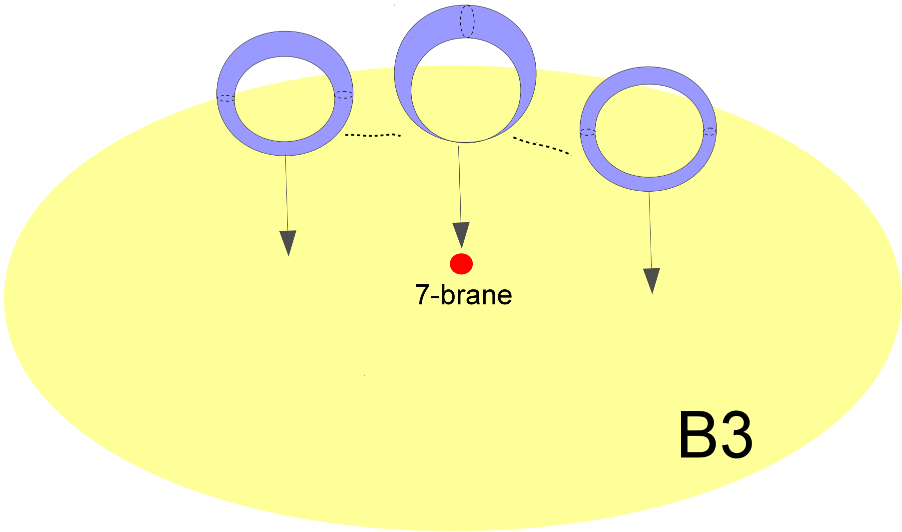


Figure 2.1: The points in the manifold where the fibre becomes singular corresponds to the position of a 7-brane in the threefold base. B_3.

which is the behaviour we observed in section 2.2 of the axio-dilaton near a D7-brane. We can therefore interpret the vanishing locus of Δ as hypersurface wrapped by a D7-brane in the geometry. Note that a simple zero of Δ signifies as D7-brane and leaves the full geometry smooth but we can in general have multiple zeros that coincide to make the full compactification space singular and correspond to a $[p, q]$ 7-brane (Figure 2.1).

One of the most attractive aspects of the F-theory framework is that the 7-branes are a result of the geometry. Unlike the case of Type IIB orientifolds where the 7-branes must be added to the geometry by hand in order to cancel the RR charge of the O7-plane and yield a consistent quantum theory, in F-theory the charge cancellation is automatically accounted for. In addition, the problems with 7-brane backreaction discussed in section 2.2 is no longer an issue. Once a consistent fibred Calabi-Yau is specified the backreaction is already built in and no additional branes have to be added. On the other hand the D3-brane tadpole condition is not automatic and boils down to a topological constraint on the compactification space.

2.3.4 A-D-E gauge symmetries

As just mentioned, there are different ways in which to the fibre can degenerate. There exists a complete classification of the types of singularities that can occur in elliptically fibred K3 manifolds due to Kodaira [9]. The type of singularity controls the resulting gauge symmetry on the brane (appendix A). If Δ factors into 24 pieces, each with distinct zeros, then each corresponding brane will have a $U(1)$ gauge symmetry. In Kodaira's classification this is called an I_1 singularity.

The details of how to get more general gauge symmetries from the A, D, and E Lie algebra series are beyond the scope of this review. It will suffice here to say that there are standard techniques in algebraic geometry to resolve the singularities of manifold by blowing up two-cycles at the point of the singularity. These two-cycles then intersect each other and the intersection points correspond to the nodes of the affine Dynkin diagram of the corresponding gauge group. The number of linearly independent two-cycles then corresponds to the rank of the gauge group. The singular limit of the manifold is then when these non-contractible two-cycles shrink to zero size and we can trace the origin of the gauge symmetry to the collision of these zero sized two-cycles in the manifold. The results can be summarised in the vanishing order of Δ, f, and g as in Table 2.1. This classification shows that the maximum singularity that can be achieved is to E_8. Singularities that are worse than this lead to technical issues that can destroy the Calabi-Yau property of the manifold.

The reader may wonder how to interpret the exceptional gauge symmetries in the framework of open strings stretching between branes. Using the analysis of appendix A it was impossible construct such groups. It was shown in [11] that it is possible to use the open string picture if we allow for multi-pronged string junctions. The analysis relies on the existence of mutually non-local 7-branes of different $[p, q]$ type. Recall that a $\binom{p}{q}$-string may end on a $[r, s]$-brane provided $(p, q) = \pm(r, s)$, where the minus corresponds to opposite orientation. It

ord(f)	ord(g)	ord(Δ)	fiber type	singularity type
≥ 0	≥ 0	0	smooth	none
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8

Table 2.1: Kodaira classification of A-D-E fibre singularities taken from [10].

is also true that a $\binom{p}{q}$-string can end on an $\binom{r}{s}$-string, and vica-versa, provided $ps - qr = \pm 1$. We say such strings are compatible. This is a result of performing an SL$(2, \mathbb{Z})$ transformation on the well know result that an F1 can end on a D1.

To see how a string junction is formed we consider the case of an $\binom{r}{s}$-string looping around a $[p, q]$-brane on which it cannot end, i.e. $ps - qr \neq \pm 1$. As the string crosses the branch cut a $\binom{p}{q}$-string is formed by the SL$(2, \mathbb{Z})$ monodromy of the brane . If, however, the $\binom{r}{s}$- and $\binom{p}{q}$-strings are compatible a $\binom{p}{q}$-string can form to connect the brane and the $\binom{r}{s}$-string if the latter crosses the brane (Figure 2.2) . We now have a triple-pronged string with one end on the $[p, q]$-brane. The forming of the extra prong is analogous the Hanany-Witten effect of crossing p-branes [12]. The extra generators needed to form exceptional Lie algebras come from the multiple prongs of the string junction ending on mutually non-local 7-branes in the base manifold. Explicit constructions of E_6, E_7, and E_8 in this framework can be found in [11].

2.3.5 Tate's algorithm

We have seen that Kodaira's classification of singular fibres can be used to determine the gauge symmetry on the brane in the case of elliptic $K3$. We would, however, like to generalise this method to other Calabi-Yau manifolds with possi-

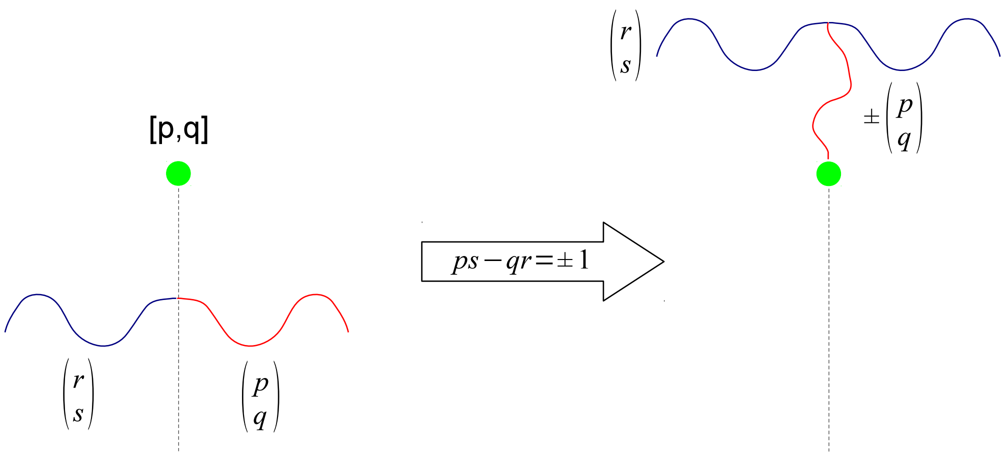


Figure 2.2: The forming of a 3 pronged string junction as the string crosses the brane. The dashed line is the branch cut due to the brane. Adapted from [11].

bly higher dimension. Such a method is provided by Tate's algorithm [13] which was first discussed in the context of F-theory in [10]. It allows the gauge group to be determined for general Calabi-Yau manifolds without the need for an explicit resolution of the singularity.

In order to describe Tate's algorithm it is convenient to generalize the Weierstrass form to the so called Tate form

$$P_W = x^3 - y^2 + a_1 xyz + a_2 x^2 z^2 + a_3 y z^3 + a_4 x z^4 + a_6 z^6 = 0, \tag{2.20}$$

which, using (2.10) to set $z = 1$, we write as

$$P_W = x^3 - y^2 + a_1 xy + a_2 x^2 + a_3 y + a_4 x + a_6 = 0. \tag{2.21}$$

The a_i are sections of sections of the line bundle $\mathcal{L}^{\otimes i}$ (this notation will be reused for other coefficients defined in this section). The equivalence to the Weierstrass form (2.11) can be made by defining quantities

$$
\begin{aligned}
b_2 &= a_1^2 + 4a_2 \\
b_4 &= a_1 a_3 + 2a_4 \\
b_6 &= a_3^2 + 4a_6 \\
b_8 &= -b_2^2 b_8 - 8b_4^3 - 27b_6^2 + 9b_2 b_4 b_6,
\end{aligned}
\tag{2.22}
$$

and we then have the Weierstrass sections as

$$
\begin{aligned}
f &= -\frac{1}{48}(b_2^2 - 24b_4) \\
g &= -\frac{1}{184}(-b_2^3 + 36b_2 b_4 - 216b_6).
\end{aligned}
\tag{2.23}
$$

If $w = 0$ defines the vanishing locus of the discriminant, the algorithm proceeds by investigating the divisibility of the a_i by various powers of w. The singularities are blown up one by one and the divisibilities of the a_i are checked each time causing the result to branch into various Kodaira types. This process eventually terminates and the Kodaira type of the singularity has been determined by the order of the divisibility of the a_i. These results we summarised in [10] and are displayed in Table 2.2. As we mentioned earlier, the most general elliptic curve can be locally modelled by the Weierstrass form (2.11) which we have shown can be written in the Tate form (2.20). It is possible in some cases, however, to define at Tate form globally. In this case the gauge group can be read off Table 2.2 straight away without needing to follow the algorithm outlined in [10] to bring the model into the correct form.

As an example, an $SU(5)$ model defined on divisor $w = 0$ takes the form

$$
a_1 = \beta_5, \quad a_2 = \beta_4 w, \quad a_3 = \beta_3 w^2, \quad a_4 = \beta_2 w^3, \quad a_6 = \beta_0 w^5, \tag{2.24}
$$

where the β_i do not contain any overall w factors. The discriminant then takes

type	group	a_1	a_2	a_3	a_4	a_6	Δ
I_0	—	0	0	0	0	0	0
I_1	—	0	0	1	1	1	1
I_2	$SU(2)$	0	0	1	1	2	2
I_3^{ns}	unconven.	0	0	2	2	3	3
I_3^s	unconven.	0	1	1	2	3	3
I_{2k}^{ns}	$Sp(k)$	0	0	k	k	$2k$	$2k$
I_{2k}^s	$SU(2k)$	0	1	k	k	$2k$	$2k$
I_{2k+1}^{ns}	unconven.	0	0	$k+1$	$k+1$	$2k+1$	$2k+1$
I_{2k+1}^s	$SU(2k+1)$	0	1	k	$k+1$	$2k+1$	$2k+1$
II	—	1	1	1	1	1	2
III	$SU(2)$	1	1	1	1	2	3
IV^{ns}	unconven.	1	1	1	2	2	4
IV^s	$SU(3)$	1	1	1	2	3	4
$I_0^{*\,ns}$	G_2	1	1	2	2	3	6
$I_0^{*\,ss}$	$SO(7)$	1	1	2	2	4	6
$I_0^{*\,s}$	$SO(8)^*$	1	1	2	2	4	6
$I_1^{*\,ns}$	$SO(9)$	1	1	2	3	4	7
$I_1^{*\,s}$	$SO(10)$	1	1	2	3	5	7
$I_2^{*\,ns}$	$SO(11)$	1	1	3	3	5	8
$I_2^{*\,s}$	$SO(12)^*$	1	1	3	3	5	8
$I_{2k-3}^{*\,ns}$	$SO(4k+1)$	1	1	k	$k+1$	$2k$	$2k+3$
$I_{2k-3}^{*\,s}$	$SO(4k+2)$	1	1	k	$k+1$	$2k+1$	$2k+3$
$I_{2k-2}^{*\,ns}$	$SO(4k+3)$	1	1	$k+1$	$k+1$	$2k+1$	$2k+4$
$I_{2k-2}^{*\,s}$	$SO(4k+4)^*$	1	1	$k+1$	$k+1$	$2k+1$	$2k+4$
$IV^{*\,ns}$	F_4	1	2	2	3	4	8
$IV^{*\,s}$	E_6	1	2	2	3	5	8
III^*	E_7	1	2	3	3	5	9
II^*	E_8	1	2	3	4	5	10
non-min	—	1	2	3	4	6	12

Table 2.2: Summary of the Tate's Algorithm. The Kodaira fibre and corresponding gauge group given in terms of vanishing order of coefficients of Tate form. Taken from [10].

the form

$$\Delta \sim w^5(...), \tag{2.25}$$

where the factor in brackets is generically irreducible and is manifested as an additional I_1 factor.

2.4 Gauge enhancements

In order to get the a geometric realisation of a realistic gauge theory based on the gauge group of the 7-branes we need additional ingredients. We need a geometric origin for chiral matter and the Yukawa couplings between them. As we have seen, the gauge theory on 7-brane comes from a complex codimension 1 hypersurface in the threefold base space. Chiral matter lives on a codimension 2 hypersurface defined by the intersection of 7-branes. Yukawa couplings are the points (complex codimension 3) in the base space at the intersection of three seven branes.

2.4.1 Matter curves

At the intersection of two codimension 1 fibre degeneration loci we find a codimension 2 locus of enhanced fibre singularity type. By enhanced here we mean that rank of the associated A-D-E gauge group is increased with respect to the two codimension 1 singularities. It is important to note here that although it is common practice in the literature to use experience with Tate's algorithm from codimension 1 singularities to determine the gauge group of singularities of higher codimension, this is outside the general validity of Tate's algorithm [14]. As such, the discussion of enhanced gauge symmetries in F-theory, although vital for model building, is still a conjecture.

The setup is described by two hypersurfaces, S_1 and S_2, wrapped by seven branes with gauge groups G_1 and G_2 respectively. Matter becomes trapped at the one complex dimensional curve define by their intersection (Figure 2.3),

$$S_1 \cap S_2 = \Sigma_{12}. \tag{2.26}$$

The rank of the enhanced gauge group, G_{12}, is the sum of the ranks of G_1 and G_2. Along the intersection the we obtain extra representations in order to fill out the degrees of freedom of G_{12}. Under to branching $G_{12} \to G_1 + G_2$ the adjoint of G_{12} decomposes as

$$\text{Adj}(G_{12}) \to (\text{Adj}(G_1), 1) \oplus (1, \text{Adj}(G_2)) \bigoplus_i (R_1^i, R_2^i), \tag{2.27}$$

where the R_1^i and R_2^i are non-trivial representations of G_1 and G_2 respectively. The zeros modes of the Dirac operator on Σ_{12} constitute the chiral matter of the 4D effective theory and they are contained in these additional representations [15]. This situation is familiar from D-brane model building in Type II theories where an $SU(n)$ stack intersects an $SU(m)$ stack. In this case, strings stretching between the stacks gives rise to additional bifundamental matter transforming in the $(m, \bar{n}) \oplus (\bar{m}, n)$ representation of $SU(m) \times SU(n)$. In the context of the M-theory dual, the matter localised on the curve corresponds to M2 branes wrapping two cycles that degenerate along the curve.

In order for the matter to be chiral we require non-zero 7-brane flux. The flux, F, takes a value in a subgroup of the seven brane gauge group $H \subset G$ and index theory is used to determine the net number of chiral generations along the matter curve [16, 2],

$$\#\text{ chiral modes} = \int_{\Sigma_{12}} \left(F_{S_1} + F_{S_2}\right). \tag{2.28}$$

The number of chiral generations can therefore be arranged by choosing suitable gauge flux on the seven branes. Note however that gauge flux can also break the gauge group to the commutant of H in G which can be problematic for GUT

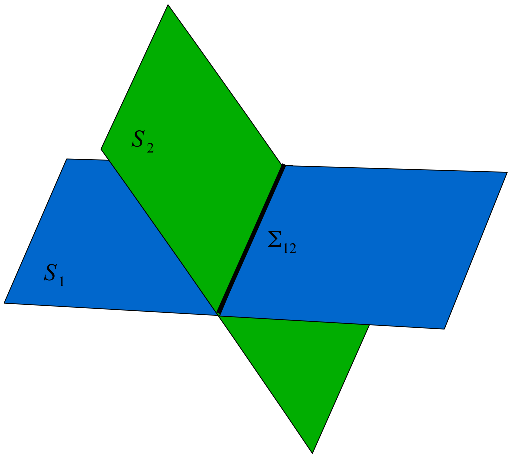


Figure 2.3: The intersection locus of two seven branes in F-theory results in an enhanced gauge symmetry that produces chiral matter trapped along the matter curve Σ_{12}.

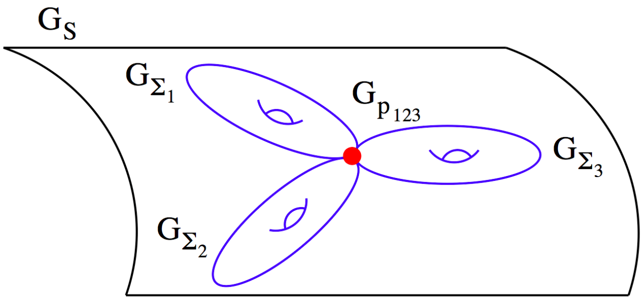


Figure 2.4: A rank 2 enhancement of the 7-brane gauge group forming at the intersection of three matter curves. Taken from [2].

model building. We will see later however that this provides a method for breaking the GUT to the Standard Model in a phenomenologically viable way.

2.4.2 Yukawa couplings

There can be a further gauge enhancement at the point of intersection of matter curves (Figure 2.4). Although it is possible to have enhancements at the intersection of two matter curves, in order to form a gauge invariant interaction three curves must intersect (we will see an example of this later when we consider an $SU(5)$ in detail). At the intersection of three matter curves Σ_{12}, Σ_{23}, and Σ_{13} combine to fill the adjoint of the gauge group G_{123}. The Yukawa interactions originate from the decomposition of the $\mathrm{Adj}(G_{123})^{\otimes 3}$, and gives a superpotential term of the form [17]

$$W \supset \lambda_{ijk} \Psi_i \Psi_j \Psi_k, \tag{2.29}$$

where the Ψ_i are the chiral superfields localised on the matter curves. The Yukawa coupling λ_{ijk} is given by the overlap of the wavefunctions in the 7-brane,

$$\lambda_{123} = \int_S \psi_1 \psi_2 \psi_3, \tag{2.30}$$

where the ψ_i are components of chiral superfields Ψ_i.

Chapter 3

F-Theory GUTs

In this chapter we will embed a GUT model into the framework of F-theory introduced in the previous chapter. Recent years have seen much success in this area with the works [2, 18, 16] sparking much interest in the field.

Before we can consider any example of a GUT model in F-theory we must choose a gauge group. In chapter 1 we introduced the $SU(5)$ model but we did not give any detail on GUTs based on higher rank gauge groups such as $SO(10)$ and E_6. These groups also have some nice properties. For example the spinor representation **16** of $SO(10)$ can accommodate all of the chiral matter of the MSSM in one representation. The **16** also accounts for a right handed neutrino which can be used in a seesaw mechanism to generate small neutrino masses. It is well known however that reproducing the **16** from the perturbative Type IIB approach is impossible.

We have seen that the maximum enhancement of the gauge symmetry from fibre degeneration is to E_8. Therefore if we require rank two enhancements for Yukawa couplings then the only exceptional simple groups can contain the Standard Model are indeed $SU(5)$, $SO(10)$, and E_6. It turns out that GUT breaking and requiring the existence of a limit where gravity is decoupled causes problems for the non-minimal GUTs. In particular there is a no-go theorem for $SO(10)$ GUTs that says breaking the gauge group to the Standard Model via internal hyperflux (more later) necessarily introduces exotics to the low energy spectrum [18]. For the E_6 there are similar problems that require very fine tuned geometries to eliminate exotics. It is for this reason that most of the F-theory literature has been focused on $SU(5)$ models and here we will treat them exclusively.

3.1 Decoupling limit

The philosophy behind local models is motivated by the large difference is energy scales between the GUT scale and the Planck scale. This suggests that we should be able to treat only the gauge degrees of freedom initially and postpone consideration of gravitational effects. The decoupling limit $M_{pl} \to \infty$ is not absolutely necessary but is sensible from the point of view of GUT physics for which UV completeness requires this limit to exist.

In order to see how this limit constrains the geometry we must relate the 4 dimensional Planck scale and the GUT scale with geometrical parameters. If we compactify the Einstein-Hilbert action on the threefold base B_3 we find

$$S_{EH} = M_*^8 \int_{\mathbb{R}^{1,3} \times B_3} d^{10}x \sqrt{-g} R \tag{3.1}$$

where in M_* is a fundamental scale given by l_s^{-1} in the IIB limit. We therefore have that

$$M_{pl}^2 = M_*^8 Vol(B_3). \tag{3.2}$$

The GUT scale however is set by the volume of the 2 complex dimensional Kähler surface, S, wrapped by the 7-brane

$$M_{GUT}^{-4} \simeq Vol(S). \tag{3.3}$$

We see then that the decoupling limit is determined by the relative volume of B_3 and S. For $M_{pl} \to \infty$ we require

$$Vol(S) \to 0, \qquad Vol(B_3) \text{ finite.} \tag{3.4}$$

This requires S to be Fano [18]. The 2 complex dimensional Fano surfaces are the del Pezzo surfaces $\mathbb{P}^1 \times \mathbb{P}^1$, $\mathbb{P}^2$, and dP_N, $N = 1, ..., 8$. The dP_N are $\mathbb{P}^2$'s with N two-cycles blown up. We see that the existence of a decoupling limit allows only a short list of possibilities for S and so it is quite restrictive. We will see later that this also greatly restricts the possibilities for GUT breaking mechanism. The existence of such a surface wrapped by a 7-brane in the compactification space is one of the most constraining ingredients in F-theory model building and can lead to some degree of predictivity. Explicit construction of a family F-theory compactifications on elliptically fibred Calabi-Yau fourfolds that admit such a decoupling can be found in [19].

3.2 $SU(5)$ model

We start with an $SU(5)$ gauge symmetry from the vanishing of a discriminant of the form (2.25). This describes an $SU(5)$ gauge theory on a 7-brane which we will assume wraps a del Pezzo surface. This is the brane on which all gauge degrees of freedom localise and we will call it the GUT brane. In order to introduce chiral matter and Yukawa couplings we proceed as in section 2.4 by considering rank 1 and rank 2 gauge enhancements. The allowable gauge enhancements can be obtained by considering the Dynkin diagram of $SU(5)/A_4$ and adding one and two nodes to create other A-D-E Dynkin diagrams. The resulting gauge groups are shown in Figure 3.1.

These enhancements occur due to intersections with other 7-branes in the

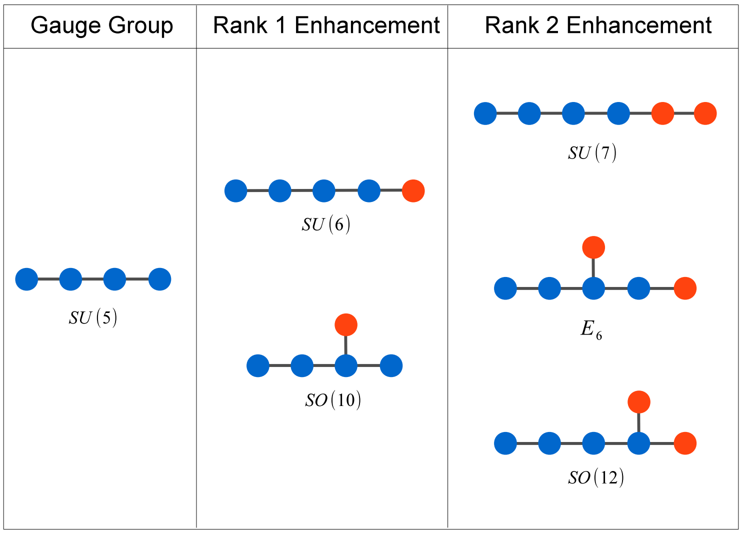


Figure 3.1: The allowable rank 1 and rank 2 A-D-E gauge enhancements of $SU(5)$ obtained by adding nodes the the A_4 Dynkin diagram.

compactification space. For example, the additional I_1 locus of (2.25) results in a $U(1)$ 7-brane that can intersect the GUT brane leading the a rank 1 enhancement. As we have seen, rank 1 enhancements lead to chiral matter and so this $U(1)$ brane will be called the matter brane. To find the type matter produced we decomposed the adjoint of the enhanced gauge groups into representations of $SU(5)$ and $U(1)$. Under the breaking $SO(10) \to SU(5) \times U(1)$ the **45** of the $SO(10)$ enhancement decomposes as [20]

$$\mathbf{45} \to \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{10}_2 \oplus \overline{\mathbf{10}}_{-2}. \tag{3.5}$$

The $SO(10)$ enhancement therefore produces the **10** matter of the $SU(5)$ GUT. The **5** is similarly obtained from the $SU(6)$ enhancement

$$\mathbf{35} \to \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_1 \oplus \overline{\mathbf{5}}_{-1}. \tag{3.6}$$

As an aside we note that we may use this procedure to produce the **16** of $SO(10)$ that descends from an $E_6 \supset SO(10) \times U(1)$ enhancement

$$\mathbf{78} \to \mathbf{45}_0 \oplus \mathbf{1}_0 \oplus \mathbf{16}_{-3} \oplus \overline{\mathbf{16}}_3. \tag{3.7}$$

This illustrates the importance of taking a non-perturbative approach to string model building.

In section 2.4 we saw that in order engineer chirality we need to turn on 7-brane flux (see (2.28)). At this stage we do not want to consider breaking the GUT so we ensure that there is no flux on the GUT brane. Instead we turn on the abelian flux on the matter brane that intersects the GUT brane along a matter curve. This can be tuned to ensure we get the 3 chiral generations required for the Standard Model.

It order to reproduce the necessary Yukawa interactions in (1.5) we must consider rank 2 gauge enhancements. These will reduce to the Yukawa interactions of the MSSM (1.3) after GUT breaking. If we first consider the decomposition of the adjoint of E_6 under $E_6 \to SU(5) \times U(1)^2$ we find [20]

$$\mathbf{78} \to \mathbf{24}_{0,0} \oplus \mathbf{1}_{0,2} \oplus \mathbf{1}_{0,0} \oplus \mathbf{5}_{6,0} \oplus \mathbf{10}_{-3,1} \oplus \mathbf{10}_{-3,-1} \oplus \text{c.c.} \tag{3.8}$$

The Yukawa coupling then descend from the cubic term in the 7-brane Chern-Simons action and we find the interaction

$$\mathbf{78}^3 \supset \mathbf{5}_H \times \mathbf{10}_m \times \mathbf{10}_m. \tag{3.9}$$

This interaction will yield masses for the up-type quarks of the MSSM after electroweak symmetry breaking. It is important to note here that Type II model building it has been a challenge to include this as a non-zero interaction because it is forbidden by $U(1)$ symmetries. It is necessary to include D-brane instanton corrections in order to achieve it [21]. This need for non-perturbative effects is reflected in the F-theory approach which requires the E_6 enhancement to generate the coupling. This again suggests that viable phenomenological models are best described in an inherently non-perturbative framework such as F-theory.

If we conduct a similar analysis for the $SO(12)$ and $SU(7)$ enhancements we find that the $SO(12)$ adjoint decomposes as

$$\mathbf{66} \to \mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{5}_{2,2} \oplus \mathbf{5}_{-2,2} \oplus \mathbf{10}_{0,4} \oplus \text{c.c.} \tag{3.10}$$

This yields the down-type quark Yukawa

$$\mathbf{66}^3 \supset \overline{\mathbf{5}}_H \times \overline{\mathbf{5}}_m \times \mathbf{10}_m. \tag{3.11}$$

We therefore have reproduced the desired Yukawas of the four dimensional $SU(5)$ GUT (1.5). The $SU(7)$ adjoint decomposes as

$$\mathbf{48} \to \mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{5}_{0,6} \oplus \mathbf{5}_{-7,1} \oplus \mathbf{1}_{7,5} \oplus \text{c.c.} \tag{3.12}$$

This can be used to generated neutrino masses via the coupling

$$\mathbf{48}^3 \supset \mathbf{5}_H \times \overline{\mathbf{5}}_m \times \mathbf{1}_{N_R}. \tag{3.13}$$

Here the $\mathbf{1}_{N_R}$ is a right-handed neutrino. It is a GUT singlet and its matter curve is perpendicular to GUT brane and intersects the $\mathbf{5}_H$ and $\overline{\mathbf{5}}_m$ curves only at the $SU(7)$ point. We will discuss neutrinos further in chapter 5.

The gauge symmetry of the rank 2 enhancement points is Higgsed to lower symmetry along the matter curves. The gauge symmetry of the matter curves is then further Higgsed to the GUT symmetry of the brane. For example the in the E_6 case we have E_6 locally Higgsed to the $SU(6)$ of the $\mathbf{5}$ matter curve and Higgs curve, and the $SO(10)$ of the $\mathbf{10}$ matter curve. These are further Higgsed to $SU(5)$ on the brane. We can see from (3.8) that each of the factors in the $\mathbf{5}_H \times \mathbf{10}_m \times \mathbf{10}_m$ interaction carry different $U(1)^2$ charges. Each matter factor must therefore localise on different matter curves and intersect only at the enhancement point. Local cancellation of the $U(1)^2$ charges at this point is the reason we need precisely three curves to intersect at this point.

3.3 Point of E_8 and spectral covers

The requirement of $SO(12)$, E_6, and $SU(7)$ enhancement points along with constraints from constructing realistic quark and neutrino hierarchies suggests a further embedding of these structures into a single point of E_8 [22, 23]. All other interactions then descend from this point. Recall that E_8 is the maximum allowable singularity if we require the manifold to remain Calabi-Yau. If we consider again the $SU(5)$ Tate form

$$y^2 = x^3 + \beta_5 xy + \beta_4 w + \beta_3 w^2 y + \beta_2 w^3 x + \beta_0 w^5 \tag{3.14}$$

we can see that this is just the E_8 singularity

$$y^2 = x^3 + w^5 \tag{3.15}$$

deformed or unfolded down to $SU(5)$. By sequentially smoothing out the deformation by letting the $\beta_i \to 0$ we recover $SO(10)$, E_6, E_7, and E_8 singularities [24].

In this way we can think of an underlying E_8 gauge symmetry that exists everywhere on the divisor wrapped by the GUT brane. To reproduce our earlier picture we imagine this E_8 deformed to various degrees throughout the divisor. This suggests that the model can be described solely in terms of these deformations.

If we consider the E_8 symmetry as resulting from a stack on branes in the strongly coupled Type IIB language then the deformations are encoded in the adjoint valued scalar field, Φ, that lives on the stack. The variation of this Higgs field over the stack then determines the gauge symmetries, and hence the matter and Yukawa couplings, of the setup. In this way the model can be locally described as a Higgs bundle [24]. In the spectral cover approach the Higgs bundle is replaced with just the eigenvalues of Φ at each point of the divisor.

As an example we can consider giving a vev to Φ by blowing up some of the two-cycles of the E_8 (or in a brane picture folding away some of the branes) to produce and $SU(5)$ gauge symmetry

$$E_8 \to SU(5) \times SU(5)_\perp. \tag{3.16}$$

The Higgs field Φ then takes values in $\mathrm{Adj}(SU(5)_\perp)$ and its eigenvalues μ_i, $i = 1,..,5$ are the roots of the equation $\det(w\mathbb{1}_5 - \Phi) = 0$ and are stored in the spectral cover. Under this breaking pattern the adjoint of E_8 decomposes into representations of $SU(5) \times SU(5)_\perp$ as

$$\mathbf{248} \to (\mathbf{24}, 1) \oplus (1, \mathbf{24}) \oplus (\mathbf{5}, \mathbf{10}) \oplus (\overline{\mathbf{5}}, \overline{\mathbf{10}}) \oplus (\mathbf{10}, \overline{\mathbf{5}}) \oplus (\overline{\mathbf{10}}, \mathbf{5}). \tag{3.17}$$

We therefore see the appearance of the required matter curves on the stack on branes. In Higgs bundle language the $SU(5)_\perp$ is the holonomy group of the Higgs bundle.

The usefulness of the spectral cover approach really becomes apparent when describing gauge flux for GUT breaking. The allowed spectral covers that described the necessary gauge flux while also meeting various phenomenological constraints are highly constrained (see e.g. [25]).

Chapter 4

GUT breaking and SUSY breaking

In order to make contact with the low energy physics of the MSSM it is necessary to break the GUT group. In this chapter we will discuss what options are available to us in the most phenomenologically attractive F-theory compactifications. The method of GUT breaking has many consequences that can upset the phenomenology (e.g. proton decay) so we discuss some of these. It turns out that consistent GUT breaking is one of the most restrictive aspects of F-theory model building. We will also discuss how supersymmetry breaking can be embedded in the F-theory framework.

4.1 Mechanism of GUT breaking

It turns out that the requirement of a gravitational decoupling limit severely restricts the mechanisms for GUT breaking. One traditional approach is the break the GUT by giving a non-zero vev to the adjoint valued holomorphic $(2, 0)$ form Higgs field, Φ. This method is familiar from gauge theories. If our GUT brane wraps a del Pezzo surface (as is required for decoupling), however, the zero mode content does not contain any holomorphic $(2,0)$ form chiral superfields to play the role of the Higgs since $h^{2,0}(S) = 0$. In the brane picture we can understand this as result of the absence of gravity preventing branes from moving in the compactification space to yield the geometric Higgsing. This is also a dynamical breaking mechanism and so requires a suitable breaking potential which can be challenging to arrange. As was noted in [26] models that break the GUT via a Higgs mechanism end up similar to four-dimensional GUT models which have many issues with phenomenological constraints.

Another traditional approach to breaking the GUT group in string compactifications is by turning on discrete Wilson lines. This is a non-dynamical breaking because it is already built into the topological data of the compactification. This however requires and non-trivial fundamental group, π_1, on which to wrap the 1-cycles. This is again unavailable to us because all del Pezzo surfaces have trivial fundamental groups.

There is however an third option available. As we mentioned earlier it is possible to break the GUT by turning on $U(1)$ fluxes on the GUT brane [18, 26]. This is again an non-dynamical breaking. We will focus on the $SU(5)$ model because, as we mentioned earlier, breaking non-minimal GUTs with internal flux necessarily generates exotics in the low energy spectrum. In this case we can give a non-zero vev to the flux of hypercharge (hyperflux), $U(1)_Y$, to break $SU(5)$ down to the commutant

$$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{4.1}$$

The hypercharge generator, T_Y, embeds in $SU(5)$ as $T_Y = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2})$.

The $SU(5)$ representations then decompose as [26]

$$\begin{aligned}
\mathbf{24} &\to (\mathbf{8},1)_0 \oplus (1,\mathbf{3})_0 \oplus (1,1)_0 \oplus (\overline{\mathbf{3}},\mathbf{2})_{\frac{5}{6}} \oplus (\mathbf{3},\mathbf{2})_{-\frac{5}{6}}, \\
\mathbf{10} &\to (\mathbf{3},\mathbf{2})_{\frac{1}{6}} \oplus (1,1)_{-1} \oplus (\overline{\mathbf{3}},1)_{\frac{2}{3}}, \\
\overline{\mathbf{5}} &\to (\overline{\mathbf{3}},1)_{-\frac{1}{3}} \oplus (1,\mathbf{2})_{-\frac{1}{2}}.
\end{aligned} \tag{4.2}$$

In other string compactifications, such as those of the heterotic string, non-trivial internal flux can cause the photon to develop mass via the Stuckleberg mechanism. This is a result of coupling of the Yang-Mills theory on the brane to closed string modes in the bulk via the seven brane Chern-Simons interaction [3]

$$S_{\text{Stuckleberg}} \simeq \int_{\mathbb{R}^{1,3}} F_Y^{4D} \wedge c_2^i \ \text{tr}T_Y^2 \int_S c_1(L_Y) \wedge \iota^* \omega_i. \tag{4.3}$$

Here the ω_i provide a basis for the two-forms in the base manifold B, $H^2(B,\mathbb{Z})$, and the self-dual four form of IIB has be decomposed as $C_4 = c_2^i \wedge \omega_i$. The hyperflux data has been packaged into a non-trivial line bundle, L_Y, with first chern class, $c_1(L_Y)$, that is Poincaré dual to $\beta \in H_2(S)$. Finally, ι^* denotes the pullback map of the embedding $\iota : S \to B$. We therefore see that in order to avoid generating a mass for $U(1)_Y$ we must only have non-trivial flux on the two-cycle β that is non-trivial in S but lifts to trivial cycle in B i.e. there exists a three-chain, α in B such that $\beta = \partial\alpha$. The two-cycle β is then said to lie in the relative cohomology of S in B. This considerably restricts the choice of hyperflux in F-theory compactifications. It is impossible to avoid such a mass in heterotic compactifications with non-trivial internal flux and so when we consider these setups in F-theory the duality with the heterotic string [7] is lost.

4.2 Additional phenomenological constraints

The allowable hyperfluxes that can be use to break the GUT to the MSSM must overcome several phenomenological obstacles of which we will mention a few.

4.2.1 No non-MSSM exotics

The decomposition of the adjoint **24** of $SU(5)$ after GUT breaking produces the $SU(3)_C \times SU(2)_L \times U(1)_Y$ representations $(\overline{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}} \oplus (\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$ (see (4.2)). There is no matter in the MSSM that transforms in these representations so they must somehow be lifted from the low energy spectrum. Since representations (sometimes referred to as XY bosons) descend from the adjoint they propagate in the full bulk of the GUT brane. Such states are count by the cohomology groups of the hyperflux, L_Y^q, where q is the hypercharge of the state. It was shown in [18] that by considering the vanishing of such fractional powers of line bundles that these exotics can be avoided. This is non-trivial to arrange and reduces the freedom in choosing hyperflux considerably. As we mentioned earlier, this step is impossible in many non-minimal GUTs that necessarily have low energy exotics in the spectrum if the GUT is broken by hyperflux [18]. This is one of the main reasons that the minimal $SU(5)$ model has received the most attention in the literature.

In (4.2) we also see that the decomposition of $\overline{\mathbf{5}}$ produces the representation $(\overline{\mathbf{3}}, 1)_{-\frac{1}{3}}$. While this is needed in the chiral matter curve for the right-handed down-type quark, on the Higgs curve these triplets should not be in the low energy spectrum. Since the component representations of the **5** curve have different hypercharges it is possible to lift one of the components from the low energy spectrum if there is a non-zero net hyperflux penetrating the curve. This is not desirable for the case of the chiral matter because we want to retain full GUT multiplets. We must therefore arrange that the net hyperflux through the matter curves is zero. If, however, the Higgs curve is separated from the matter curve and the hyperflux through it is non-zero then the Higgs triplets can acquire a large mass. The hyperflux can then also cause the Higgs up, $\mathbf{5}_H$, and Higgs down, $\overline{\mathbf{5}}_H$, curves to can also split due to the their opposite hypercharge. We will see in a moment that this is the preferred situation for suppressing proton decay.

4.2.2 Proton decay

Proton decay in in the 4d effective theory can be caused by dimension four, five, and six operators [27]. Dimension 6 operators can cause proton decay through the channel $p \to e^+\pi^0$. These take the form

$$\frac{\alpha_6}{M_{GUT}} \int d^4\theta \; u_R^\dagger e_R^\dagger QQ \tag{4.4}$$

and are mediated by the off-diagonal XY bosons of the GUT. Fortunately these operators are suppressed by the GUT scale and are currently safely outside current experimental bounds. It is, however, important to ensure that dimension four and five operators, which can give rise to experimentally unacceptable decay rates, are suppressed in a natural way in our model.

Dimension four operators descend from the Yukawa interaction $\mathbf{10}_m \times \overline{\mathbf{5}}_m \times \overline{\mathbf{5}}_m$ and so includes terms like $u_R d_R d_R$, $u_R d_R L$, and $e_R LL$. These operators are forbidden by R-parity in the MSSM. R-parity is $\mathbb{Z}_2$ symmetry under which SM matter has parity +1 and its superpartners has parity -1. In terms of chiral superfields we have chiral matter superfields with parity -1 and Higgs matter superfields with parity +1. In order to form an R-parity invariant cubic interaction we therefore need the Higgs superfield. In F-theory we want to find a geometric origin for this. It was shown in [18] that this is achievable if we the Calabi-Yau fourfold has a $\mathbb{Z}_2$ reflection symmetry. All the line bundles, which encode matter curve and flux data, must have a definite parity under this $\mathbb{Z}_2$. The hyperflux must also have a definite parity and if it is to be integrated over a matter curve then the result will only be non-zero if the curve and the hyperflux have the same parity. We have already seen that the we require zero net hyperflux through the chiral matter curves and non-zero through the Higgs curves. This discrete difference distinguishes the parity of the Higgs and chiral matter curves and so forbids the troublesome dimension four operators.

From the perspective of global models base on E_8 spectral covers, it is shown in [28] that dimension four proton decay can be forbidden using a global $U(1)_{PQ}$ symmetry. Such a symmetry arise naturally when E_8 is unfolded to $SU(5)$ and the charge assignments for the various matter representations prevents both the $\mathbf{10}_m \times \overline{\mathbf{5}}_m \times \overline{\mathbf{5}}_m$ Yukawa and a tree-level μ-term, $\sim \mu H_u H_d$.

Dimension 5 operators such as

$$\frac{\alpha_5}{M_{GUT}} \int d^4\theta \, QQQL \qquad \text{and} \qquad \frac{\alpha_5'}{M_{GUT}} \int d^4\theta \, u_R d_R u_R e_R \tag{4.5}$$

are mediated by the exchange of heavy Higgs colour triplets. While we have arranged that the Higgs triplets become very massive and so this should be quite suppressed, we must also worry about and entire Kaluza-Klein tower of states that will contribute to these operators when the triplets are integrated out. The interactions arise from the superpotential terms

$$W_{GUT} \supset QQT_u + u_R e_R T_u + QLT_d + u_R e_R T_d + M_{GUT} T_u T_d, \tag{4.6}$$

where the terms 1 and 2 originate from the up-type Yukawa ($\mathbf{10}_m \times \mathbf{10}_m \times \mathbf{5}_H$) and terms 3 and 4 originate from down-type Yukawa ($\mathbf{10}_m \times \overline{\mathbf{5}}_m \times \overline{\mathbf{5}}_H$). It was shown in [18] that these interactions could be avoided using a geometric implementation of the missing partner mechanism known to field theory. If instead of pairing with each other, T_u and T_d pair with other heavy triplets T_u' and T_d' then the problematic triplet mass term of (4.6) could be avoided. This is achieved geometrically by allowing the Higgs curve $\mathbf{5}_H \oplus \overline{\mathbf{5}}_H$ split into two parts, $\mathbf{5}_H$ and $\overline{\mathbf{5}}_H$. In our discussion

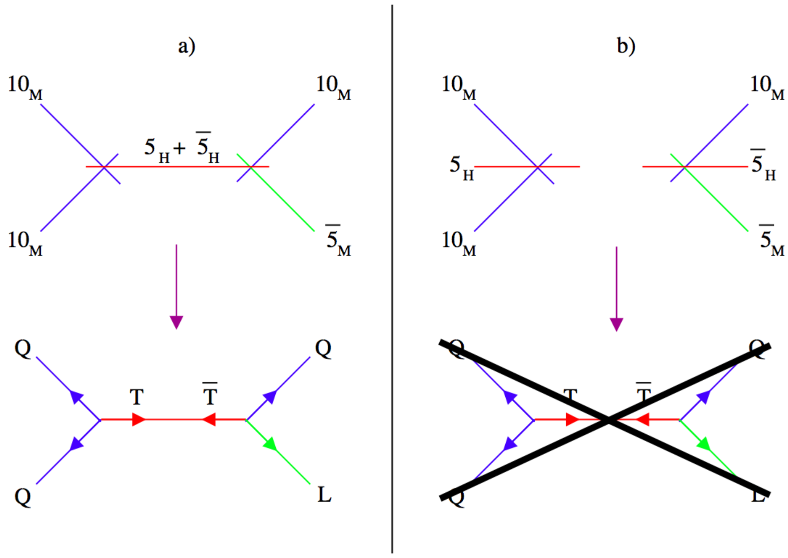


Figure 4.1: In a) Higgs up and Higgs down are on same matter curve leading to the generation of unwanted dimension 5 proton decay operators. Splitting Higgs up and down as in b) removes this possibility. Taken from [18].

of doublet-triplet splitting we already mentioned that this can be engineered by suitable choice of hyperflux. This geometric splitting can be directly interpreted as splitting of the unwanted Feynman diagram in the 4d effective theory (Figure 4.1). This can again be enforced in the spectral cover picture using $U(1)_{PQ}$.

4.2.3 Gauge coupling unification

One consequence of hyperflux GUT breaking is the alteration of the running of the MSSM coupling constants. This is not to be taken lightly because it may spoil the unification which is well founded in low energy experimental data and motivated the study of GUT structures. The GUT scale is generally define as the energy at which the couplings of $SU(2)_L$ and $U(1)_Y$ intersect. The question of whether we have full gauge coupling unification is then whether or not the $SU(3)_C$ passes through this intersection point. It was shown in [29] and [26] that the internal flux alters the gauge kinetic functions of each factor of the Standard Model through the 7-brane Chern-Simons coupling

$$S_{CS} \sim \int_{\mathbb{R}^{1,3}\times S_{GUT}} C_0 \wedge \mathrm{tr}(F), \tag{4.7}$$

where F is the total gauge flux on the seven brane. Instead of unifying the couplings satisfy [25]

$$\alpha_1^{-1} - \frac{3}{5}\alpha_3^{-1} - \frac{2}{5}\alpha_3^{-1} = 0. \tag{4.8}$$

This does not, however, explain why experimental data points towards unification. It is argued in [29] that introduction of a new scale between the weak scale and the GUT scale can correct the running of coupling constants leading to ‘F-unification’. Fortunately we already have such as scale available to us: the mass of the Higgs triplets. If we include a vector like pair of Higgs triplets, $T_u \oplus T_d$, and set their mass to lie in the range $10^{15} - 10^{16}$ GeV then unification can be retained. Above this new scale the beta-function coefficients are modified

$$(b_3, b_2, b_1) = (3, -1, -11) \qquad \rightarrow \qquad (b_3, b_2, b_1) = \left(2, -1, -\frac{35}{5}\right). \tag{4.9}$$

We therefore find that requiring doublet-triplet splitting to obtain the correct spectrum and suppress proton decay automatically gives us a solution to gauge coupling unification. In [25] it is argued that massive exotic matter from *any* incomplete GUT multiplets can be used as this new scale to correct unification.

4.3 SUSY breaking

Up to this point we have been assuming $\mathcal{N} = 1$ supersymmetry in the low energy theory. While this is well motivated and attractive from a theoretical viewpoint it disagrees with current particle physics data. Therefore if supersymmetry exists in the real world it must be broken and any theoretical model of supersymmetric particle physics must incorporate a mechanism to achieve this. To agree with experiment all the superpartners of the Standard Model matter must be given a mass to lift them from the low energy spectrum. This must be achieved in such a way that quadratic divergences are not introduced. This is known as soft supersymmetry breaking.

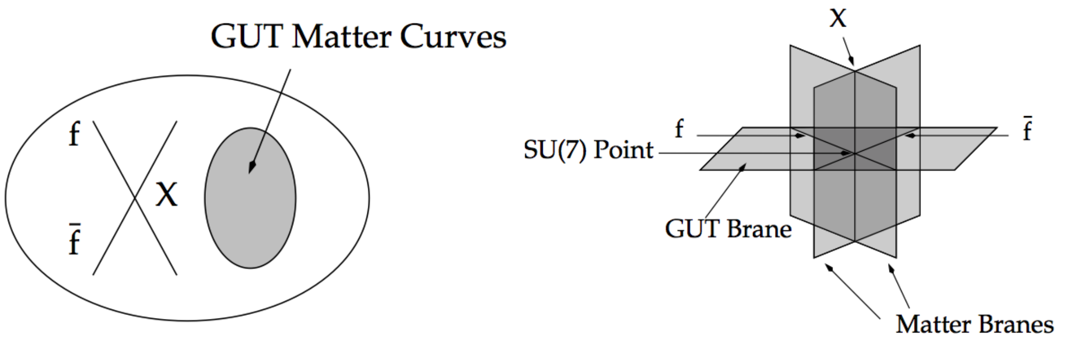
A well established method for soft supersymmetry breaking is to break supersymmetry in a hidden sector and have a messenger sector communicate this to the MSSM matter, known as the visible sector. In the hidden sector supersymmetry is broken dynamically and can be parametrised by chiral superfield, X, that has a non-zero vev

$$\langle X \rangle = x + \theta^2 F. \tag{4.10}$$

The scale of the supersymmetry breaking in the hidden sector is then set by $\sqrt{F}$.

It is important to specify the form of the messenger fields because it plays an important role in the phenomenology. The two main approaches considered are gravity mediation and gauge mediation. The primary drawback of the gravity mediation approach is the introduction of flavour changing neutral current (FCNC) processes. These flavour violating process are troublesome and often lead to theories inconsistent with experiment. In gauge mediation scenarios the FCNCs are suppressed but work has to be done to address the μ problem naturally. This issue can be resolved in many cases however which makes gauge mediation appealing. Theories with gauge mediated SUSY breaking are reviewed in [30].

A framework for gauge mediated SUSY breaking in F-theory models have been developed in [31] and [32]. In an $SU(5)$ F-theory model the messenger fields are comprised of the vector-like pairs $\mathbf{5} \oplus \overline{\mathbf{5}}$ or $\mathbf{10} \oplus \overline{\mathbf{10}}$. The X field is a GUT singlet and localises on a matter curve normal to the GUT brane. In this way the hidden sector, where SUSY breaking takes place, is on a different brane to the GUT brane. The X field then interacts with the messenger fields, f and $\overline{f}$,



(a) GUT Brane with f and $\bar{f}$ matter curves and the singlet X

(b) Depiction of the triple intersection at the $SU(7)$ enhancement point

Figure 4.2: F-theory geometry of $\mathbf{5}\oplus\overline{\mathbf{5}}$ messenger fields and SUSY breaking sector. Taken from [31].

through the superpotential term

$$W \supset \lambda X f \overline{f}. \tag{4.11}$$

When the X field has a non-zero vev as in (4.10) this will generate a mass for the messenger fields. The SUSY breaking is then communicated to the visible sector because the MSSM fields also interact with the messenger fields. The gaugino masses are generated at one loop order and the scalar masses are generated at two loop order. The geometry of this setup where the interaction (4.11) descends from an $SU(7)$ gauge enhancement is shown in Figure 4.2.

Next we must address the μ problem. The μ problem get its name from the superpotential term

$$W \supset \mu H_u H_d. \tag{4.12}$$

The natural scale for μ is the GUT scale but the electroweak symmetry breaking of the Standard Model requires it to lie at the weak scale. As we have mentioned, and explicit μ-term can be evaded in F-theory by splitting the $\mathbf{5}_H$ and $\overline{\mathbf{5}}_H$ curves using an $U(1)_{PQ}$ symmetry. The fields are charged under the $U(1)_{PQ}$ as [18]

	Φ	H_u, H_d	f, $\overline{f}$	X
$U(1)_{PQ}$	+1	-2	+2	-4

where Φ is any MSSM field other than the Higgs doublets. Clearly this symmetry forbids a GUT scale μ-term (4.12). A weak scale μ-term can then be generated by coupling the messenger fields the the Higgs. When the massive messenger fields are integrated out we obtain an effective operator of the form

$$\frac{1}{M_X}\int d^4\theta X^\dagger H_u H_d, \tag{4.13}$$

where M_X is the scale at which the operator is generated. From a geometric point of view this operator is generated at $SU(7)$ Yukawa point where X, $\mathbf{5}_H$, and $\overline{\mathbf{5}}_H$ intersect. When the X field has a non-zero vev as in (4.10) this will yield $\mu \sim \frac{\overline{F}}{M_X}$. For messenger scale just below the GUT scale we have $M_X \sim 10^{15}$ GeV so we require $F \sim 10^{17}$ GeV^2 for a weak scale μ.

Since the X field is charge under $U(1)_{PQ}$ the F-term vev of X that breaks SUSY will also break $U(1)_{PQ}$ at this scale. The massive $U(1)_{PQ}$ will then contribute additional soft term contributions beyond the minimal scenario presented above. This gives rise to a coupling between the chiral matter and X through the exchange of the heavy $U(1)_{PQ}$ [32]. This generates the operator

$$-4\pi\alpha_{PQ}\frac{e_\Phi e_X}{M^2_{U(1)_{PQ}}}\int d^4\theta X^\dagger X \Phi^\dagger \Phi, \tag{4.14}$$

where $M_{U(1)_{PQ}}$ is the mass of $U(1)_{PQ}$ and the e_i are the charges of the superfields under $U(1)_{PQ}$. This will clearly give rise to another mass contribution when X gets a vev. This setup is therefore referred to as *deformed* gauge mediated supersymmetry breaking.

As a first attempt one might not be concerned how supersymmetry is broken in the hidden sector and only require that is broken somehow and at the correct scale. However a complete model must of course address this. It was shown in [33] that dynamical SUSY breaking can be induced in the hidden sector by Euclidean D3-instantons if appropriate fluxes on the f and $\overline{f}$ matter branes are chosen. It was shown in [31] that the conditions needed to achieve this fit precisely with the gauge mediation scenario described above.

Chapter 5

Flavour and neutrinos in F-Theory

Up to this point in our treatment all three generations of MSSM matter has been identical. We arranged for three chiral matter generations by appropriate choice of gauge flux on the matter brane but there is no immediate reason why any of these generations should be different. This is of course not the case in reality. Particle physics data reveals mass hierarchies in the quark and neutrino sectors that must be explained. The different flavours of both quarks and leptons also mix non-trivially. If we hope to build a realistic model in the F-theory framework then we must reproduce these flavour textures in the compactification. Recent work has found that F-theory can naturally incorporate both quark and neutrino flavour sectors [18, 34, 35, 23].

5.1 Quark flavour

To frame the discussion we will first review what we need to reproduce. The up-type quarks fall into the mass hierarchy $(u, c, t) = (0.003, 1.3, 170)$ GeV, whereas the down-type quarks have masses $(d, s, b) = (0.005, 0.1, 4)$ GeV [1]. The leptons follow a similar structure with $(e, \mu, \tau) = (0.0005, 0.1, 1.8)$ GeV. The difference between the mass and gauge quark eigenstates is quantified in the Cabibo-Kobayashi-Maskawa (CKM) mixing matrix [1]

$$|V_{CKM}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix}. \tag{5.1}$$

There is also the well known GUT scale mass relation $m_b \sim m_\tau$. This fits well with both the b and the τ Yukawas descending from the same GUT Yukawa $\overline{\mathbf{5}}_H \times \overline{\mathbf{5}}_m \times \mathbf{10}_m$ but this cannot explain the absence of such a relations for the lighter generations.

As a first approximation for the up-type quarks we should require that one quark, the t, be hierarchically heavier that the other two generations. Subleading effects will then deform the Yukawas to create a smaller hierarchy for the lighter generations. To see how this might arise in F-theory we consider the superpotential term

$$W \supset \lambda^u_{ij}\ \mathbf{5}_H \times \mathbf{10}^{(i)}_m \times \mathbf{10}^{(j)}_m \tag{5.2}$$

To leading order the Yukawa coupling is given by the product of the wavefunctions at the point of intersection. Generically there can be more than one such intersection point which we must sum over. For example the up-type quark Yukawa is given by

$$\lambda^u_{ij} = \sum_p \psi^{H_u}(p)\psi^Q_i(p)\psi^{u_R}_j(p). \tag{5.3}$$

The Higgs term contributes a constant factor at each point but the chiral matter terms are vectors containing an entry for each generation. This is therefore and outer product of vectors yielding a 3×3 matrix. In a setup where we have three distinct curves that do not self intersect then λ^u will take the form

$$\lambda^u = \begin{pmatrix} 0 & A & B \\ A & 0 & C \\ B & C & 0 \end{pmatrix}. \tag{5.4}$$

It was shown in [18] that this setup cannot give one hierarchically heavier generation as is required for up-type quarks. In the limit of one of the mass being zero the determinant of λ^u will vanish

$$2ABC = 0. \tag{5.5}$$

We can take the solution as $A = 0$. Since the trace of λ^u vanishes the remaining eigenvalues must be equal and of opposite sign. We therefore have not obtained the desired result. Instead of one heavy and two light (approximately massless here) we have two quarks that are hierarchically heavier that the last. This is an unacceptable phenomenology and so we must reject the assumption that we have three distinct curves that do not self intersect. Instead we take the two $\mathbf{10}_m$ factors to originate from the same curve that self intersects at the Yukawa point. At first sight this seems to contradict out discussion in section 3.2 where we argued that the $U(1)^2$ charges of the matter curves required three distinct curves for charge conservation. This can be reconciled however using 7-brane monodromy [17]. The monodromy of the brane produces a branch cut and as the matter curve crosses it it is splits into two curves with different $U(1)^2$ charges. From the spectral cover viewpoint the monodromy group permutes the eigenvalues of Φ that unfold the Yukawa enhancement. Under the breaking $G_p \rightarrow G_{GUT} \times \Gamma$ the monodromy group is a subgroup of the Weyl group of Γ. If we consider the breaking pattern [17]

$$E_6 \rightarrow SU(6) \times SU(2) \rightarrow SU(5) \times U(1) \times SU(2) \tag{5.6}$$

then we get the matter

$$\mathbf{78} \rightarrow (\mathbf{24}, 1) \oplus (1, \mathbf{3}) \oplus (\mathbf{5}_{-6}, 1) \oplus (\overline{\mathbf{5}}_{6}, 1) \oplus (\mathbf{10}_{-1}, \mathbf{2}) \oplus (\overline{\mathbf{10}}_{3}, \mathbf{2}). \tag{5.7}$$

We therefore have the $\mathbf{10}_m$ curves transforming as a doublet of $SU(2)$. The Weyl group of $SU(2)$ is $\mathbb{Z}_2$ so the monodromy group just interchanges these two **10**'s. Such a setup can yield to first approximation two massless generations and one heavy generation.

The next step is to find an origin to the subleading hierarchies. A common method for achieving this in four-dimensional GUTs is introduce and additional global $U(1)$ symmetry and generate the hierarchies using the Froggatt-Nielsen mechanism [36]. In this approach it was shown that correct masses and mixing for the quark sector could be generated if the Yukawa matrices took form

$$\lambda^u_{ij} = g^u_{ij}\epsilon^{a_i+b_j} \qquad \text{and} \qquad \lambda^d_{ij} = g^d_{ij}\epsilon^{a_i+c_j}, \tag{5.8}$$

where ϵ is a small parameter. This is usually arranged for by introducing a GUT singlet charged under the additional global $U(1)$ and coupling it to the matter fields that form the Yukawa. The chiral matter fields are assigned $U(1)$ charges to form invariants when interacting with the new singlet. When this singlet admits a vev and the $U(1)$ breaks we obtain the desired form of the Yukawa matrices from the couplings

$$g^u_{ij}\left(\frac{\langle\phi\rangle}{M_{pl}}\right)^{a_i+b_j} H_u Q^i u^j_R \qquad \text{and} \qquad g^d_{ij}\left(\frac{\langle\phi\rangle}{M_{pl}}\right)^{a_i+c_j} H_d Q^i d^j_R. \tag{5.9}$$

While this will yield the desired results for clever choices of a_i, b_i, and c_i this merely sidesteps the question. It is natural to desire some physical argument for the charge assignments.

There is a natural way to incorporate Froggatt-Neilsen like symmetries in F-theory by considering the effect of background fluxes on the matter wavefunctions [18, 34]. The wavefunctions have $U(1)$ rephasing symmetry under the action of the internal Lorentz group. The three generations can by labelled by the vanished order of the wavefunctions at the Yukawa point, $\psi^i \sim z^{3-i}$, $i = 1, ..3$. Background gauge flux and three form flux can distort the wavefunctions via the Aharonov-Bohm effect and break the rephasing symmetry [34]

$$\psi \to \exp(\mathcal{M}_{i\bar{j}} z_i \bar{z}_{\bar{j}})\psi. \tag{5.10}$$

As an example we can consider the effects of hyperflux on the chiral matter wavefunctions. Earlier we enforced the condition that the net hyperflux penetrating a matter curve is zero in order to maintain full GUT multiplets for the chiral matter, and to facilitate doublet-triplet splitting and proton decay suppression. It was noted in [18] however that since the hyperflux will not necessarily vanish pointwise this can have an effect on the wavefunctions. Since the components of the GUT multiplets have different hypercharge this will effect them all differently and yield different Yukawas for each generation and matter type. It turns out that this has the least effect on the heaviest generation and so the GUT scale mass relation $m_\tau \sim m_b$ is maintained but such relations for the lighter generations break down.

Including the effects of three form flux into $\mathcal{M}$ can generate realistic quark flavour textures [34]. To calculate corrections coming from the distortion $\mathcal{M}$ it can be expanded in two different ways. Firstly it can be expand in terms of higher derivatives of the flux (DER) or in terms of higher powers of the first derivative of the flux (FLX). The contributions of the various leading order and subleading corrections to the Yukawa matrix can be summarised as [34]

$$\lambda \sim \lambda_0 + \lambda_{DER} + \lambda_{FLX} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} + \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{5.11}$$

where the first contribution comes from the self intersection of the **10** curve and so is absent for down-type Yukawas. Either λ_{DER} or λ_{FLX} will dominate for a given matter type depending on the hypercharge. For the up-type quarks λ_{FLX} dominates whereas λ_{DER} dominates for down-type quarks. The natural scale for these distortions is $\epsilon \sim \sqrt{\alpha_{GUT}} \sim 0.2$. This yields the mass hierarchies

$$\begin{aligned} m_u : m_c : m_t &\sim \epsilon^8 : \epsilon^4 : 1 \sim 0.004 : 0.8 : 170 \\ m_d : m_s : m_b &\sim \epsilon^5 : \epsilon^3 : 1 \sim 0.006 : 0.08 : 4, \end{aligned} \tag{5.12}$$

which, as can be seen from the beginning of this section, are remarkably close to the true hierarchies. Finally, the CKM matrix is given by $V_{CKM} = V_u^L V_d^{L\dagger}$. Here $V_L \sim V_R$ are the unitary matrices required to diagonalise the Yukawa matrices. This yields

$$|V_{CKM}| \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.008 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}, \tag{5.13}$$

which is again very similar to the observed result.

As a finally point on quark flavour we note that the calculation of the CKM matrix requires that hierarchy in both the up-type and down-type Yukawa manifest itself in the same basis for Q matter fields since they appear in both Yukawas. This can be arranged by ensuring the up and down Yukawa points are close to each other in the compactification space. While this introduces a mild fine tuning this can be evaded by appealing to the point of E_8 picture. In this case the up and down Yukawas originate from a single E_8 enhancement point and so coincide precisely.

5.2 Neutrinos

The observation of neutrino oscillations, solving the famous solar neutrino problem, is the first experimental evidence of particle physics beyond the Standard Model. The mixing of the three neutrino generations requires a small but non-zero mass for neutrinos of the order of 10^{-2} eV. This mass scale is far below the electroweak scale which suggests a fundamental difference between neutrino mass generation and that of the other leptons of the Standard Model. One popular approach is to generate a small Majorana mass using the seesaw mechanism which requires a right-handed neutrino with a Majorana mass near the GUT scale. This hints at a possible connection between GUTs and neutrino physics. Indeed one of the most attractive features of the non-minimal $SO(10)$ GUTs is the presence of a right-handed neutrino in the spectrum.

There are *apriori* two possibilities for the neutrino to develop a mass: a Dirac mass, or a Majorana mass. These are respectively generated from terms of the form

$$m_{ij}^{\text{Dirac}} N_L^i N_R^j \qquad \text{and} \qquad m_{ij}^{\text{Maj.}} N_L^i N_L^j, \tag{5.14}$$

where N_L denotes the left-handed neutrino superfield component of the lepton $SU(2)$ doublet, L, of the MSSM. Although the Majorana terms if forbidden by the gauge symmetries of the MSSM it can be generated using the effective higher dimensional operator

$$W \supset \lambda_{ij} \frac{(H_u L^i)(H_u L^j)}{\Lambda_{UV}}, \tag{5.15}$$

where Λ_{UV} is an energy scale close to the GUT scale. When H_u develops a vev we obtain a Majorana mass term for the left handed neutrino. This operator can be generated using the seesaw mechanism. If we have a large Majorana mass for N_R and this couples to N_L via

$$W \supset g_{ij} H_u L^i N_R^j + M_{ij} N_R^i N_R^j, \tag{5.16}$$

then when the heavy N_R fields are integrated out we obtain the operator (5.15) where the scale Λ_{UV} is set by M.

In the Dirac mass scenario the mass term arises from the Yukawa

$$W \supset \lambda_{ij} H_u L^i N_R^j. \tag{5.17}$$

This had traditionally been problematic in string models because order one Yukawa couplings are the easiest to arrange for. In order to get a neutrino mass much lower than the weak scale we must introduce a fine tuning.

It was shown in [18] that both scenarios could be accommodated naturally in the F-theory framework. Firstly, we saw in section 3.2 that GUT singlets could couple to GUT matter at an $SU(7)$ enhancement point. These singlets localise on matter curves perpendicular the brane and only intersect it at a few points. The wavefunction of the singlet can either be exponentially suppressed or enhanced at the Yukawa point depending on the local curvature of the GUT divisor and the matter curve [18]. In the case where the wavefunction is suppressed a Yukawa coupling of the right order needed for Dirac neutrino masses is realisable. This is a direct consequence of the geometry and avoids arbitrary tuning. The Majorana scenario can occur if we have and order one Yukawa coupling from an enhanced wavefunction at $SU(7)$ in addition to a large Majorana mass for N_R. This can be achieved by introducing another GUT singlet, Φ, that intersect the N_R curve to yield

$$W \supset \lambda_{ij} H_u L^i N_R^j + g_{ij} \Phi N_R^i N_R^j. \tag{5.18}$$

If Φ develops a suitably large vev we can implement a seesaw mechanism.

This Majorana setup is not completely satisfactory however. In order to accommodate a seesaw we were forced to introduce an additional singlet and now are left the responsibility of justifying its large vev. It was noted in [23] however that since the Majorana mass scale of N_R is close to the Kaluza-Klein scale then it may be possible to interpret the right-handed neutrino as a Kaluza-Klein mode. Since these modes already have the required mass we no longer need to introduce Φ or discuss its vev. One problem remains however. Since we have bifundamental matter localised on the curve the Kaluza-Klein mass term actually couples N_R with N_R^c, its CPT conjugate. In order to resolve this problem a covering theory can be developed which is later under goes a $\mathbb{Z}_2$ quotient which identifies N_R and N_R^c [23].

The participation of these Kaluza-Klein modes in interactions reduces the mass hierarchy with respect to the quark and charged lepton sectors [23]. These models prefer a *normal hierarchy* for neutrinos such that

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \epsilon^2 : \epsilon : 1. \tag{5.19}$$

It is also possible to calculated the leptonic mixing matrix $U_{PMNS} = U_L^l \cdot (U_L^\nu)^\dagger$, where $U_{R/L}^l$ and $U_{R/L}^\nu$ are used in biunitary transformations to diagonalise the mass matrices of the charge leptons and neutrinos respectively. This obtains

$$|U_{PMNS}| \sim \begin{pmatrix} U_{e_1} & \epsilon^{\frac{1}{2}} & \epsilon \\ \epsilon^{\frac{1}{2}} & U_{\mu_3} & \epsilon^{\frac{1}{2}} \\ \epsilon & \epsilon^{\frac{1}{2}} & U_{\nu_3} \end{pmatrix} \sim \begin{pmatrix} 0.87 & 0.45 & 0.2 \\ .45 & 0.77 & 0.45 \\ 0.2 & 0.45 & 0.87 \end{pmatrix}, \tag{5.20}$$

where the diagonal elements were obtained using the constraint of unitarity. This estimate is rather close to experimental data

$$|U_{PMNS}| \sim \begin{pmatrix} 0.77-0.86 & 0.50-0.63 & 0.00-0.20 \\ 0.22-0.56 & 0.44-0.73 & 0.57-0.80 \\ 0.21-0.55 & 0.40-0.71 & 0.59-0.82 \end{pmatrix}. \tag{5.21}$$

In order to achieve this hierarchy it was necessary to unify all of the Yukawa interactions into a single point of E_8 which, as we have mentioned several times, arises naturally in F-theory. This evades a problem with the $SU(7)$ Kaluza-Klein

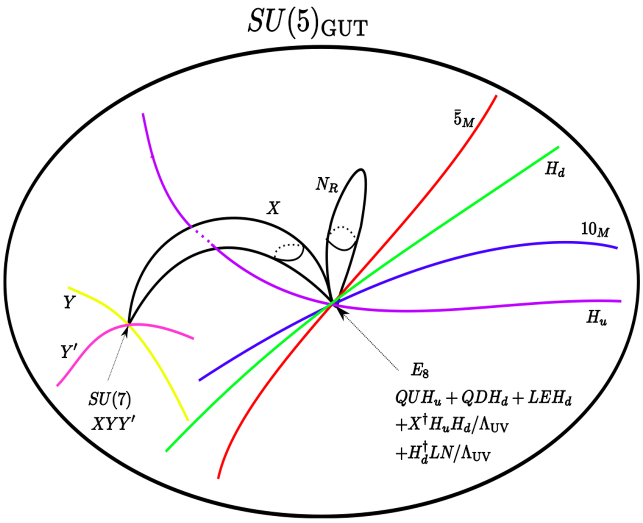


Figure 5.1: Unification of all $SU(5)$ GUT interaction into a single point of E_8. Taken from [23].

seesaw that requires the H_u and L matter curves to be identified after the $\mathbb{Z}_2$ quotient. The entire picture of a minimal F-theory GUT in this setup in shown in Figure 5.1.

At the time of the writing of [23] there seemed to be growing experimental and theoretical evidence that the U_{13} entry be precisely zero. The non-zero value was however unavoidable in the F-theory model and could therefore be deemed a prediction. Earlier this year the Daya Bay neutrino experiment reported an result of ~ 0.154 for U_{13} confirming that is non-zero [37]. While this F-theory result is far from precise it is an encouraging sign for string model building.

Conclusion

We have introduced the subject of F-theory and developed some of its application to GUT model building. The result is an appealing framework for interpreting many particle physics structures as a direct result of the geometry of spacetime. We have seen that various phenomenological considerations, such as the decoupling limit and GUT breaking, have introduced many constraints on F-theory constructions. In particular we have seen the inclusion of hyperflux play a key role in many aspects of the phenomenology. Finally, we saw that F-theory models can reproduce realistic flavour textures for the quark and neutrino sectors.

There is still, however, much work to be done in the area of F-theory phenomenology. Firstly there is ongoing work attempting to incorporate all of the ingredients of the local model into an explicit global compactification. It is possible that there may be obstructions to some of the features of the local models when considered on the global setting.

It is also important to consider the cosmological implications of such models. Although we have not dealt with this issue here, in order to construct a model that fits with cosmological data it is necessary to dynamically stabilise any compactification moduli that could affect early universe dynamics.

Appendix A

Branes, gauge theories, and root systems

To understand how a gauge theory may live on the world volume of a brane we must recall the definition of a D-brane: a hypersurface on which a fundamental string may end. From the perspective on the world volume of the D-brane the end of the string appears as a point particle. As is well known from ordinary 4d electromagnetism a point particle couples electrically to a one form field, A_μ. If we a have a Dp-brane we can used Wigners little group on the brane, $SO(p-1)$, to determine that A_μ contributes $p-1$ bosonic degrees of freedom living on the brane. We must also consider that the location of the brane in spacetime spontaneously breaks Poincaré invariance. As a result we will have a Goldstone boson for each broken generator. Since the brane is localise in $9-p$ spacetime dimensions then we will have $9-p$ Goldstone modes. These adjoint valued scalars also live on the brane and can be viewed as embedding functions of the brane in spacetime. We therefore have a total of 8 bosonic degrees of freedom on every Dp-brane. This will break supersymmetry to produce 8 fermionic Goldstino modes. D-branes are therefore $\frac{1}{2}$ BPS objects with supersymmetric $U(1)$ gauge theory on their world volume.

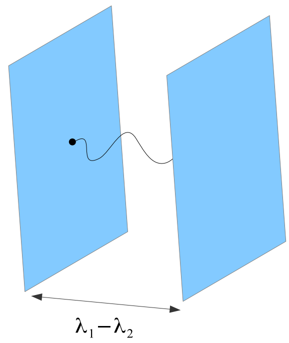


Figure A.1: Two separated D-branes with $U(1)$ connected my a massive W-boson string mode.

By introducing more branes we can furnish the degrees of freedom of more complicated groups by allowing strings to stretch between the branes. As a first example consider two D-branes with a string stretching between them. The brane positions, λ_1 and λ_2, are labelled by the eigenvalues of the scalar living on the branes. The setup can be seen in Figure A.1. When the branes are separated we have a $U(1)^2$ gauge symmetry and the string stretching between the brane constitutes a W-boson of mass $m^2 \sim (\lambda_1 - \lambda_2)^2$. When the length of the string collapses to zero size the W-boson becomes massless and the gauge symmetry is enhanced to $U(1)^2 \subset U(2) = SU(2) \times U(1)$. The $U(1)$ factor then parametrizes the position of the centre of mass of the brane system. Since the W-boson carries no charge with respect to this $U(1)$ the physics decouples from it and we are left with and $SU(2)$ gauge symmetry. We can make this relationship between stacks of branes and gauge symmetries more precise by considering arbitrary numbers of branes and strings and their relationship to root systems.

A.1 A_n root system

If we have N parallel D-branes we can have strings stretching between each brane as shown in Figure A.2. If we labelling each brane $i = 1, ..., N$ we can creating an orthonormal basis or string vectors $e_i = (0, ..., 1, ...0)$ with the 1 in the ith position

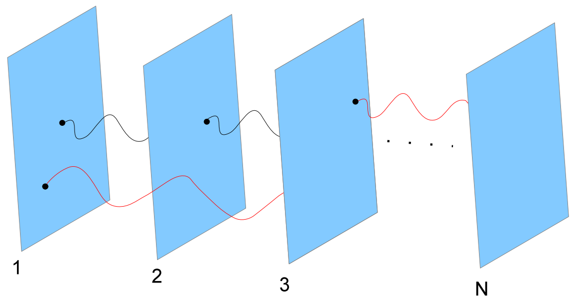


Figure A.2: Brane construction of the A_{N+1} root system. N parallel D-branes with strings stretching between them. Black strings connecting adjacent branes correspond to simple roots whereas red string are sums of simple roots i.e. positive roots.

such that $(e_i, e_j) = \delta_{ij}$. When the distance between the branes is zero this setup corresponds precisely to the root system of the A_{N+1} algebra. The simple roots are given by $\alpha_i = e_i - e_j$, $i < j$ and correspond the to black string joining adjacent branes in Figure A.2. The positive roots are given by sums of these simple roots and corresponds to the red strings. The negative roots are then just strings with the opposite orientations. We therefore have a clear geometric picture of how the gauge symmetries arise in the case of $SU(N)$.

We can immediately see that the adjoint Higgs mechanism can be geometrically interpreted as the separation of branes from the stack. In this case we obtain some non-zero mass W-bosons corresponding the strings stretching between the stacks and we break $SU(N) \to SU(M) \times SU(N-M)$.

To get other classical Lie algebras from this setup we must include orientifold planes, the fixed plane under the action of an orientifold on the spacetime.

A.2 D_n root system

In order to get D_n algebras for $SO(2n)$ with need n D-branes and an orientifold plane with negative RR charge, O^-. The negative charge means that a string can-

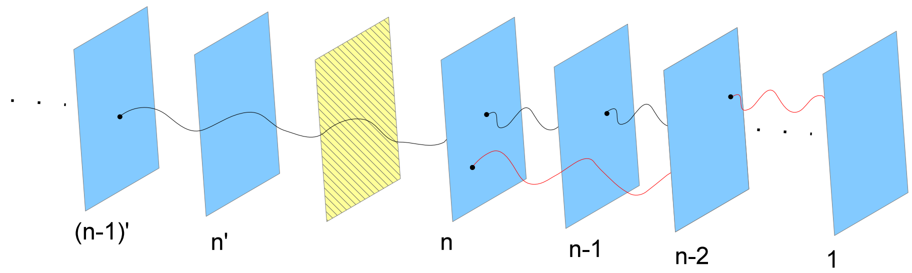


Figure A.3: Brane construction of the D_n root system. An O^- plane and n parallel D-branes with strings stretching between them. A string cannot stretch between a brane and its image so all simple roots have same length.

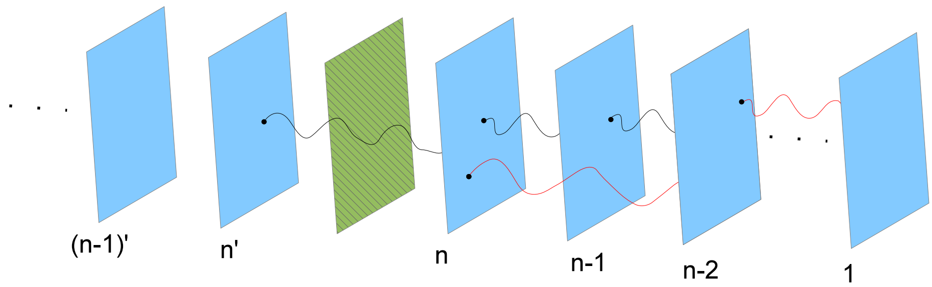


Figure A.4: Brane construction of the C_n root system. An O^+ plane and n parallel D-branes with strings stretching between them. A long root is obtained by stretching a string between n and n'.

not be stretched between a brane and its image under the orientifold. We therefore obtain the setup of Figure A.3. While the diagram depicts strings stretching between branes and images, the physical string does not cross the orientifold so we have all string of the same length. The simple roots are now given by $\alpha_i = e_i - e_j$ with $i < j$, $i = 1, ..., n-1$ and $\alpha_n = e_{n-1} + e_n$ as required for a D_n algebra. We see that Dynkin diagram is reflected in the fact that three strings constituting simple roots can attach to the $n-1$ brane (from the n, $n-2$, and n' branes), whereas only two can attach to the others.

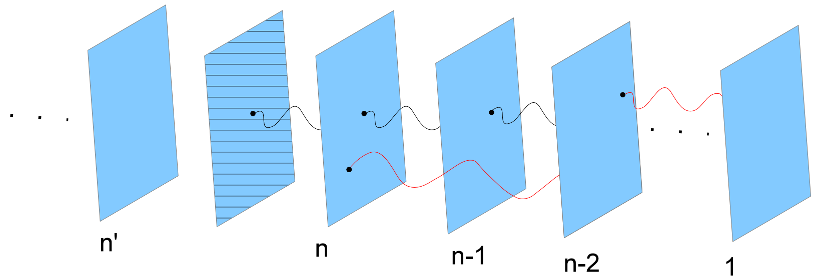


Figure A.5: Brane construction of the B_n root system. An $\tilde{O}^+$ plane and n parallel D-branes with strings stretching between them. A short root is obtained by stretching a string between n and $\tilde{O}^+$.

A.3 C_n root system

The C_n algebra can be realised using a system of n D-branes with an orientifold plane with positive RR charge, O^+. This differs from the D_n case because we can now stretch strings between a brane and its image and so we can obtain a longer root by stretching a string between n and n' (Figure A.4). The simple roots of C_n are given by $\alpha_i = e_i - e_j$ with $i < j$, $i = 1, ..., n-1$ and $\alpha_n = 2e_n$.

A.4 B_n root system

Finally for the B_n algebra we must introduce a new type of orientifold plane, the $\tilde{O}^+$. This is like an O^+ plane with half a D-brane on top of it. This construction allows strings to end on the $\tilde{O}^+$ plane to produce the configuration of Figure A.5. The resulting positive roots are given by $\alpha_i = e_i - e_j$ with $i < j$, $i = 1, ..., n-1$ and $\alpha_n = e_n$.

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